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APPROACH TO ON-LINE SYSTEM
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Chi-Shun Chao

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A Frequency Domain Based Approach to On-Line System Identification

by

Chi-Shun Chao
Lieutenant, R.O.C. Navy
B.S., Chung Cheng Institute of Technology, 1985

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of the requirements for the degree of

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ABSTRACT

This thesis addresses the problem of identifying the dynamics of a linear system in the frequency domain. An alogrithm operating on the Fast Fourier Transform (FFT) of blocks of signals is developed and its performance evaluated through computer simulations. Several properties are tested, in particular, its convergence and its capabilities of identifying the frequency response of the unknown system.

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I. INTRODUCTION

The problem of identifying the dynamics of a physical system from its input-output behavior is a fundamental issue in modeling and control. The most general approach is based on the assumption of a mathematical model, often linear and time invariant, and its parameters are determined by optimization of an error criterion.

A wide class of identification algorithms exists in the literature. Several books have been written on this matter, see for example [Ref. 1, 2]. Of particular interest are the recursive algorithms, where the estimated parameters are updated on-line based on new input-output data collected at each sampling instant. In most of the approaches on recursive identification the parameter estimates are updated for every new set of data points; as a consequence, the estimate at any time is updated up to the current data.

A different approach is based on Block Processing (BP), where the parameters are recursively updated on the basis of the data within nonoverlapping intervals of time. Although this introduces a delay between the time we measure the input-output data and the time we use it in the estimation, it has the advantage that time averaging within the time interval reduces the effect of disturbances.

Block Processing techniques have been investigated by several authors. Of particular interest for this thesis is the approach developed by Mansour [Ref. 3] in the adaptive filtering contest, where the parameters are updated on the basis of the DFT (Discrete Fourier Transform) of the data within each time interval.

In this thesis an adaptive identification algorithm operating on the DFT (Discrete Fourier Transform) of blocks of input and output data is introduced. In particular, the algorithm identifies the frequency response of the system's linear model by applying several recursive estimation techniques and adapting them to the frequency domain approach. We will investigate estimation methods based on the Projection Algorithm and Recursive Least Squares [Ref. 4] and the results will be compared. Even though Recursive Least Squares is a complex and time consuming method, it is preferable when accuracy and speed of convergence are needed.

This thesis is organized as follows. Chapter II introduces the concept of Block Processing and develops two algorithms for recursive identification in the frequency domain. Proof of convergence is also part of this chapter. Simulation studies are introduced in Chapter III showing the effectiveness of the algorithms and the conclusion is in Chapter IV.

II. IDENTIFICATION OF LINEAR SYSTEMS

A. STATEMENT OF THE PROBLEM

Consider an LTI (Linear Time Invariant) system with input, output signals given by u(t) and y(t) respectively. The problem we address is the estimation of the parameters of a linear model in order to predict future values of the output y given past measurements of the input and output signals. The general block diagram is given in figure 2.1. The block z^{-1} denotes time delay by one clock pulse.

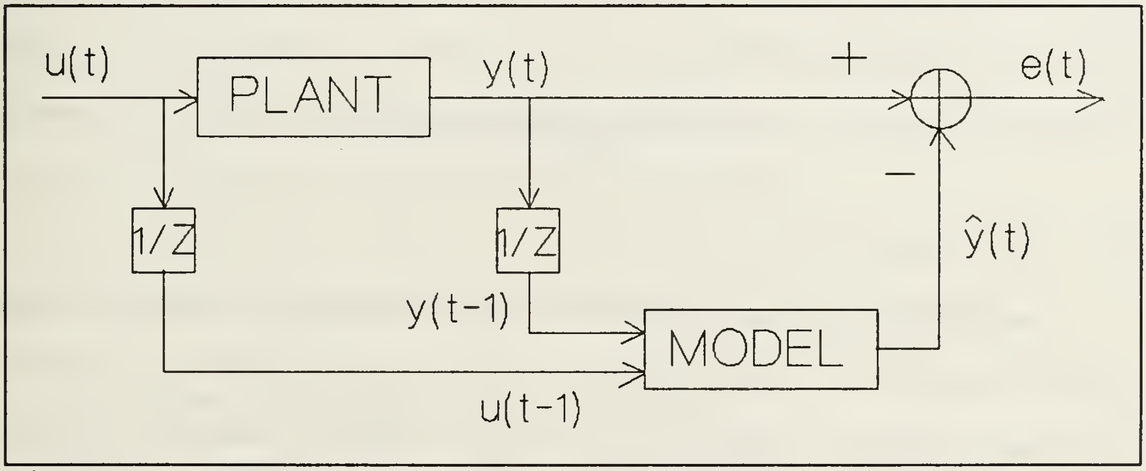

Figure 2.1 Block diagram of the system

This figure shows the general scheme where $\hat{y}(t)$ is the prediction of y(t) based on the past input, output, and the estimated model. The purpose is to determine a model for the plant so that the error e(t) is miniminized. In particular, the model is restricted to be an LTI system in either one of the two forms:

ARMA (Auto Regressive Moving Average)

$$y(t)=a_1y(t-1)+...+a_ny(t-n)+b_1u(t-1)+...b_nu(t-n) \quad (2.1)$$

where a_i, b_i are constants; or

CONVOLUTION form, where the output input signals are related by the convolution sum.

$$y(t)=h(t)*u(t) \quad (2.2)$$

$$y(t)=\sum_{\tau=-\infty}^{\infty} h(\tau)u(t-\tau) \quad (2.3)$$

The problem of adaptive frequency response identification mainly has been concentrated on the approximation of infinite impulse response systems by finite impulse response models. In this thesis, we will consider the identification of stable plants only, for which the impulse response h(t) decays to zero as time increases. For this reason, the infinite impulse response h(t) can be approximated by a finite sequence with length N. In this way, the moving average (MA) model of the following form is obtained.

$$y(t)=h(0)u(t)+h(1)u(t-1)+\cdots+h(N-1)u(t-(N-1)) \quad (2.4)$$

where h(t), for $0 \le t \le N-1$, is the truncated impulse response, We choose the block length N to be a power of 2 in order to apply FFT (Fast Fourier Transform) techniques.

The approach we consider is based on a frequency response framework. In particular, the estimated model is computed from the Fourier Transform of blocks of data collected from measurements of the input and output.

There are several reasons which motivate this approach. In many cases of interest, a model which is accurate on a desired frequency range only is identified for the plant. For example, if we want to model the low frequency spectrum of the system with the frequency response approach, we select weights in order to select frequencies of interest.

In the next section, a recursive algorithm for the identificaton of the frequency response is introduced.

B. RECURSIVE FREQUENCY DOMAIN APPROACH

Block Processing (BP) techniques for on line identification of linear models have been investigated by several authors [Ref. 3]. With this approach the estimated model parameters are sequentially updated on the basis of blocks of data, rather than at each data point. In this section, we investigate a BP adaptive algorithm which operates on the Fast Fourier Transform (FFT) of blocks of input and output data.

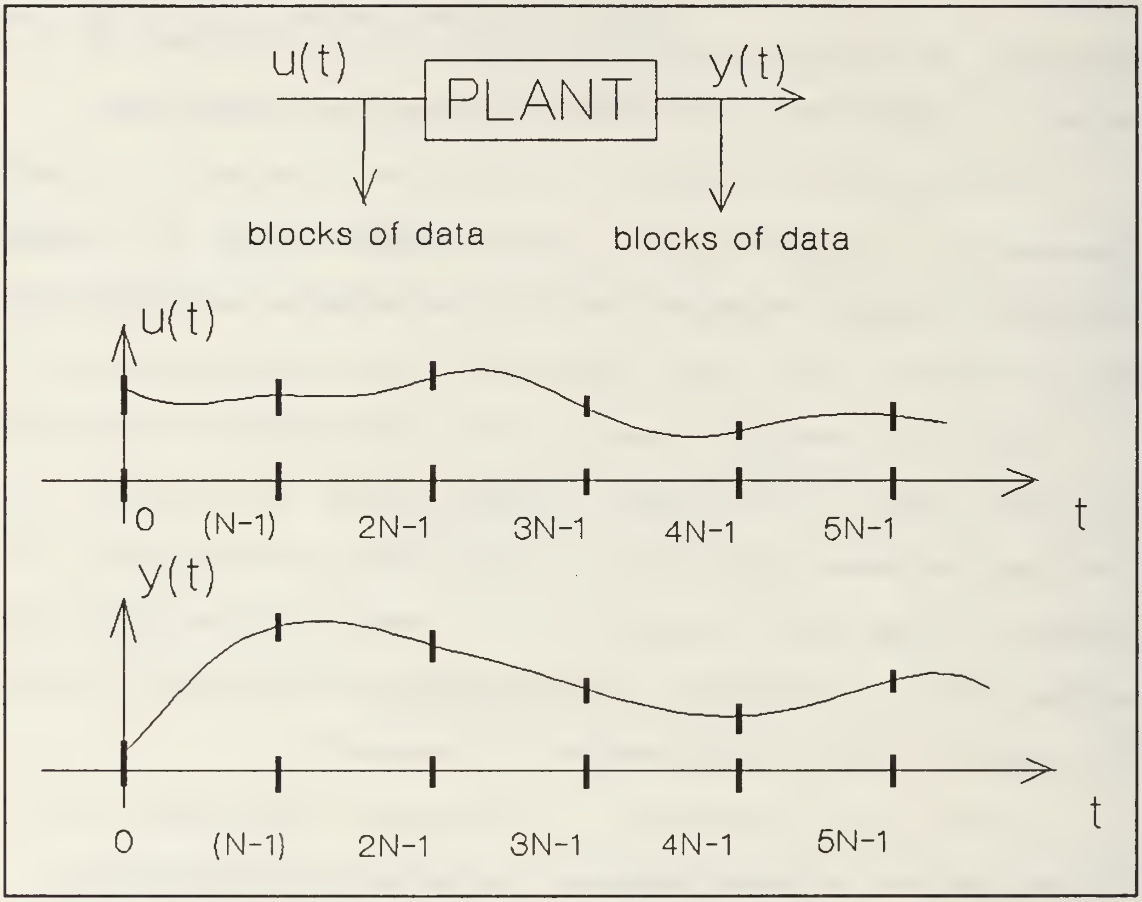

Figure 2.2 Blocks of Input and Output Data

There are some advantages in dealing with data by blocks rather than individually. Block processing averages the data within the block, resulting in better noise rejecton. In order to illustrate the BP approach in the ARMA model case, consider the model in (2.1) and the data $y_k(t)$, $u_k(t)$ within the k-th time block, i.e.

$$y_k(t) = y(kN+t)$$

$$u_k(t)=u(kN+t) \qquad (2.5)$$

for t = 0, 1, ... , N-1. If the block size N is larger than the system order n, we can write (2.1) as

$$y_k(t)=a_1y_k(t-1)+a_2y_k(t-2)+b_1u_k(t-1)+b_2u_k(t-2) \qquad (2.6)$$

for a second order case with $2 \le t \le N-1$. It is easy to see that (2.6) can be written in convolution form as

$$y_k(t)=a(t)\otimes y_k(t)+b(t)\otimes u_k(t) \qquad n\le t\le N-1 \qquad (2.7)$$

with a and b sequences in R^n

$$a=[0 \;\; a_1 \;\; a_2 \;\; \cdots \;\; a_n \;\; 0 \;\; \cdots \;\; 0]$$

$$b=[0 \;\; b_1 \;\; b_2 \;\; \cdots \;\; b_n \;\; 0 \;\; \cdots \;\; 0] \qquad (2.8)$$

and $\otimes$ denoting circular convolution.

In the approach, the well known relationship between the DFT and the circular convolution in the frequency domain is exploited. By defining { $Y_k(l)$, l=0,1, ...,N-1 } and { $U_k(l)$, l=0,1, ...,N-1 } as the DFT of the output and input data $y_k(t)$,$u_k(t)$ in the k-th block, we can define $\tilde{Y}_k(l)$ as

$$\tilde{Y}_k(l)=A(l)Y_k(l)+B(l)U_k(l) \qquad l=0,1,\cdots,N-1. \qquad (2.9)$$

Clearly from (2.7) and (2.9), we can write the important relationship

$$y_k(t)=\tilde{y}_k(t) \qquad n \le t \le N-1 \tag{2.10}$$

with $\tilde{y}_k(t)$ = IDFT[$\tilde{Y}_k(l)$], and IDFT denoting inverse DFT.

A linear estimation algorithm for A(l) and B(l) in (2.9) can be determined by defining an error signal

$$\tilde{e}_k(t)=\begin{cases} y_k(t)-\hat{y}_k(t) & n \le t \le N-1 \\ 0 & 0 \le t \le n-1 \end{cases} \tag{2.11}$$

where $\hat{y}_k(t)$ = IDFT[$\hat{A}_k(l)Y_k(l) + \hat{B}_k(l)U_k(l)$, l=0,1,...,N-1] and $\hat{A}_k$, $\hat{B}_k$ are estimates available at the k-th block. From the above definitions, we can write the recursive estimates for $\hat{A}$, $\hat{B}$ by the algorithm. Let

$$\hat{\Theta}_k(l)=\left[\hat{A}_k(l),\hat{B}_k(l)\right]^T$$

$$X_k(l)=\left[Y_k(l),U_k(l)\right]^T \tag{2.12}$$

for l = 0, 1, ..., N-1, k = 0, 1, 2, ...; also let

$$\tilde{E}_k(l)=DFT\left[\tilde{e}_k(t)\right] \tag{2.13}$$

with $\tilde{e}_k$ as in (2.11). Then the recursion is such that

$$\hat{\Theta}_{k+1}(l) = \hat{\Theta}_k(l) + \frac{X_k^*(l)\,\tilde{E}_k(l)}{1 + \max_l (|U_k(l)|^2 + |Y_k(l)|^2)} \qquad (2.14)$$

For this algorithm we can show the following:

(a) $\| \hat{\theta}_{k+1}(l) - \theta(l) \|^2 \leq \| \hat{\theta}_k(l) - \theta(l) \|^2$ (2.15)

for all l = 0, 1, 2, ..., N-1, and for all k;

(b) If the input and output data $u_k(t)$, $y_k(t)$ are bounded for all k and t, then

$$\lim_{k \to \infty} e_k(t) = 0 \qquad \textit{for all } 0 \leq t \leq N-1 \qquad (2.16)$$

In this result we see that the parameter error decreases with time (eqn. (2.14)) and the error between the model and the plant $e_k(t)$ tends to zero, i.e. the model output tends to follow the plant output.

Proof: Let us define the parameter error

$$\tilde{\Theta}_k(l) = \hat{\Theta}_k(l) - \Theta_k(l) \qquad (2.17)$$

Then from (2.14) and (2.17) we can write the recursion

$$\tilde{\Theta}_{k+1}(l) = \tilde{\Theta}_k(l) - \mu_k (X_k^*(l)\,\tilde{E}_k(l)) \qquad (2.18)$$

where

$$\mu_k = \frac{1}{1 + \max_l (|U_k(l)|^2 + |Y_k(l)|^2)} \qquad (2.19)$$

By taking the magnitude square of both sides of (2.18), we obtain

$$\|\tilde{\Theta}_{k+1}(l)\|^2=\tilde{\Theta}^{*T}_{k+1}(l)\tilde{\Theta}_{k+1}(l)$$

$$=[\tilde{\Theta}^{*T}{}_k(l)-\mu_k\tilde{E}^{*T}{}_k(l)X_k^T(l)]\,[\tilde{\Theta}_k(l)-\mu_kX_k^*(l)\tilde{E}_k(l)]$$

$$=\|\tilde{\Theta}_k(l)\|^2-\mu_k\tilde{E}_k^{*T}(l)X_k^T(l)\tilde{\Theta}_k(l)-\mu_k\Theta_k^{*T}(l)X_k^*(l)\tilde{E}_k(l)$$

$$+\mu_k^2\tilde{E}_k^{*T}X_k^T(l)X_k^*(l)\tilde{E}_k(l) \qquad (2.20)$$

By taking the sum of all frequency components in both sides of (2.20), we obtain the expression

$$\sum_{l=0}^{N-1}\|\tilde{\Theta}_{k+1}(l)\|^2=\sum_{l=0}^{N-1}\|\tilde{\Theta}_k(l)\|^2-\mu_k\sum_{l=0}^{N-1}\tilde{E}_k^{*T}(l)X_k^T(l)\tilde{\Theta}_k(l)$$

$$-\mu_k\sum_{l=0}^{N-1}\tilde{\Theta}_k^{*T}(l)X_k^*(l)\tilde{E}_k(l)$$

$$+\mu_k\sum_{l=0}^{N-1}\mu_k\tilde{E}_k^{*T}(l)X_k^T(l)X_k^*(l)\tilde{E}_k(l) \qquad (2.21)$$

Applying Parseval's theorem, we can relate data in the frequency domain to the corresponding time domain as

$$\sum_{l=0}^{N-1} [\tilde{E}_x^{*T}(l)]\,[X_k^T(l)\tilde{\Theta}_k(l)] = \sum_{t=0}^{N-1} \tilde{e}_k(t)\,\varepsilon_k(t) \tag{2.22}$$

$$\sum_{t=0}^{N-1} \tilde{e}_k(t)\,\varepsilon_k(t) = \sum_{t=2}^{N-1} |\tilde{e}_k(t)|^2 \geq 0 \tag{2.23}$$

where we define

$$\varepsilon_k(t) = IDFT[X_k^T(l)\tilde{\Theta}_k(l)] \tag{2.24}$$

Equation (2.23) comes from the fact that

$$\tilde{e}_k(t) = 0 \qquad 0 \leq t \leq n-1 \tag{2.25}$$

$$\tilde{e}_k(t) = y_k(t) - \hat{y}_k(t) = e_k(t) \qquad n \leq t \leq N-1 \tag{2.26}$$

and

$$\varepsilon_k(t) = e_k(t) \qquad n \leq t \leq N-1 \tag{2.27}$$

Still using Parseval's theorem, we can write

$$\sum_{l=0}^{N-1} \|\tilde{\Theta}_k(l)\|^2 = \|\tilde{\theta}_k\|^2 \tag{2.28}$$

After some manipulations and the fact that $\mu_k|X_k(l)|^2 \leq 1$ for all k and l = 0, 1, ..., N-1, we can bound the parameter error at the end of the (k+1)th block as

$$\|\tilde{\Theta}_{k+1}\|^2 \leq \|\tilde{\Theta}_k\|^2 - 2\mu_k\|\tilde{e}_k\| + \mu_k\|\tilde{e}_k\|^2 \qquad (2.29)$$

and therefore

$$\|\tilde{\Theta}_{k+1}\|^2 \leq \|\tilde{\Theta}_k\|^2 - \mu_k\|\tilde{e}_k\|^2 \qquad (2.30)$$

Finally, since $\| \tilde{\Theta}_k \| \geq 0$ for all k and is a nonincreasing sequence as in (2.30), the increment $\mu_k\| \tilde{e}_k\|^2$ must tend to zero as k tends to infinity. Therefore

$$\lim_{k\to\infty} \frac{\|\tilde{e}_k\|^2}{1+MAX[|Y_k(l)|^2+|U_k(l)|^2]} = 0 \qquad (2.31)$$

which proves the result.

The significance of this result is that we can estimate the parameters of the given system (in terms of $\hat{A}_k(l)$, $\hat{B}_k(l)$) based on the frequency content of each block of data. Furthermore, the estimation is recursive, and the convergence of the prediction error e(t) to zero is guaranteed at least in the ideal case. It will be shown in the simulations that even in the presence of measurement noise, the convergence of the error to small values is still satisfactory.

In the implementation of the estimation algorithm introduced above, we need to compute the DFT of the "windowed" error term $\tilde{e}_k(t)$ in (2.25) and (2.26). We can see that in the implementation two operations of IDFT are required to compute $\hat{y}_k(t)$. Using DFT again, $\tilde{E}_k(l)$ is obtained, which is used in the recursion (2.14).

An algorithm, which does not require this sequence of IDFT and DFT in its implemention, is introduced next. The new algorithm also offers the advantage of being able to use different recursive estimation techniques (such as the Recursive Least Squares) which exhibits faster convergence. However, this is obtained at the expense of added complexity.

In order to introduce the technique, let us recall the signal

$$\hat{y}_k(t)=a(t)\otimes y_k(t)+b(t)\otimes u_k(t) \qquad (2.32)$$

with a, b being the plant parameters, and the equality

$$y_k(t)=\hat{y}_k(t) \qquad n\leq t\leq N-1 \qquad (2.33)$$

Going to vector notation, let us write (2.33) in the following form

$$\overline{y}_k=\begin{bmatrix}0\\ \vdots\\ y_k(n)\\ \vdots\\ y_k(N-2)\\ y_k(N-1)\end{bmatrix}=\begin{bmatrix}[0] & [0]\\ [0] & [I]\end{bmatrix}\begin{bmatrix}\hat{y}_k(0)\\ \vdots\\ \hat{y}_k(n)\\ \vdots\\ \hat{y}_k(N-2)\\ \hat{y}_k(N-1)\end{bmatrix}=h\hat{y}_k \tag{2.34}$$

where [0] and [I] denote blocks of zeros and the identity matrix respectively. Definition of the Fourier matrix F as

$$F_{m,n}=e^{-j(\frac{2\pi mn}{N})} \tag{2.35}$$

allows the computation of the DFT of a sequence as a matrix operation. In this way, we obtain from (2.34)

$$F\overline{y}_k=Fh\hat{y}_k \tag{2.36}$$

and therefore, after simple manipulations, the following equation,

$$F\overline{y}_k=(FhF^{-1})(F\hat{y}_k) \tag{2.37}$$

It is easy to see that the vectors $F\bar{y}_k$ and $F\hat{y}_k$ are the arrays of DFT coefficients of the respective sequences $\bar{y}_k$ and $\hat{y}_k$, which yields

$$\overline{Y}_k=H\hat{Y}_k \tag{2.38}$$

Recall from (2.32) that

$$\hat{Y}_k(l) = \hat{A}_k(l)\, Y_k(l) + \hat{B}_k(l)\, U_k(l) \qquad (2.39)$$

for l=0, 1, 2, ..., N-1. Let us define YU as the following expression

$$YU = \begin{bmatrix} Y_k(0) & 0 & \cdots & 0 & \vdots & U_k(0) & 0 & \cdots & 0 \\ 0 & Y_k(1) & \cdots & 0 & \vdots & 0 & U_k(1) & \cdots & 0 \\ \vdots & 0 & \ddots & \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & Y_k(N-1) & \vdots & 0 & \cdots & 0 & U_k(N-1) \end{bmatrix} \qquad (2.40)$$

and combine (2.38) and (2.39) to yield

$$\overline{Y}_k = (HYU) \begin{bmatrix} A_k(0) \\ \vdots \\ A_k(N-1) \\ \cdots \\ B_k(0) \\ \vdots \\ B_k(N-1) \end{bmatrix} \qquad (2.41)$$

The recursive estimation algorithm is derived from (2.41). In particular, we know that (2.41) can be broken into a sequence of linear equations of the form

$$\overline{Y}_k(l) = \Phi_k^T(l)\,\Theta \qquad l=0,1,2,...,N-1 \qquad (2.42)$$

where

$$\Theta=[A_k(0)\ \cdots\ A_k(N-1)\ \ B_k(0)\ \cdots\ B_k(N-1)]^T \qquad (2.43)$$

is the vector of unknown parameters, and

$$\begin{aligned}\Phi_k^T(l) &= [H(l,0)\ \cdots\ H(l,N-1)]\,(YU)\\ &= [H(l,0)\,Y_k(0)\ \cdots\ H(l,N-1)\,Y_k(N-1)\\ &\quad ,H(l,0)\,U_k(0)\ \cdots\ H(l,N-1)\,U_k(N-1)]\end{aligned} \qquad (2.44)$$

In (2.44) the coefficient H(i,j) are constant (in the sense that they do not depend on the block k) and are determined by the window matrix h only.

From (2.42) we can use any recursive algorithm to estimate θ. In particular, a Recursive Least Squares [Ref. 2] algorithm applied to complex data is going to yield the following recursion

$$\Theta_k^{l+1}=\Theta_k^l+\frac{P_k^l\Phi_k(l)\,(\overline{Y}_k(l)-\Phi_k^T(l)\,\Theta_k^l)}{1+\Phi_k^T(l)\,P_k^l\Phi_k(l)} \qquad (2.45)$$

$$P_k^{l+1}=P_k^l-\frac{P_k^l\Phi_k(l)\,\Phi_k^T(l)\,P_k^l}{1+\Phi_k^T(l)\,P_k^l\Phi_k(l)} \tag{2.46}$$

for l = 0, 1, 2, ..., N-1, with initial conditions at the beginning of each time block given by

$$\hat{\Theta}_k^0=\hat{\Theta}_{k-1}^N \quad ; \quad P_k^0=\sigma^2 I \tag{2.47}$$

with I the identity matrix and σ^2 an arbitrary constant parameter. In general, the constant σ is chosen to be a large value, in comparison with the order of magnitude of the parameters θ.

Although far more complicated to implement, the algorithm (2.45), (2.46), and (2.47) has better convergence properties than the one shown previously.

In the next section the two algorithms are applied to the identification of a linear time invariant system.

III. SIMULATION STUDIES

A. INTRODUCTION

The estimation techniques discussed in the previous chapter have been simulated, and used to estimate the frequency response of a system. In addition, two computer programs written in MATLAB have been developed. The first one is the main program given in Appendix A which simulates the entire system in an iterative fashion. The main program estimates the value of the frequency response of a given system at a particular frequency and compares it with the corresponding value of the original system.

The second program is a function type of subroutine which implements the Recursive Least Squares algorithm [Ref. 2].

The proposed identification method has been tested using several different inputs in the program in order to simulate and analyze the effect of various excitaton conditions. In the next section, a first order recursive difference equation is used as an example.

B. SIMPLE DERIVATION

The system being used as an example is a first order system described by the difference equation.

$$y(t)=0.3y(t-1)+5.0u(t-1) \qquad (3.1)$$

In like manner (3.1) can also be written in terms of the impulse response,

$$y(t)=h(0)u(0)+h(1)u(t-1)+...=\sum_{m=-\infty}^{\infty} h(m)u(t-m) \qquad (3.2)$$

Since the system is stable, its impulse response decays to zero, and (3.2) can be approximated by a finite sum

$$y(t)=h(0)u(t)+h(1)u(t-1)+...+h(N-1)u(t-N+1) \qquad (3.3)$$

In the followig simulations N is chosen as N = 32 data points. The transfer function H(z) of the original system

$$H(z)=\frac{Y(z)}{U(z)}=\frac{5}{z-0.3} \qquad (3.4)$$

yields its frequency response

$$H(e^{j\theta})=\frac{5}{e^{j\theta}-0.3} \qquad (3.5)$$

Due to the use of FFT methods, we estimate the frequency response at discrete frequency points $\Theta = 2\pi l/N$, $l = 0, 1, 2, ..., N-1$. For this purpose, let us define

$$H(l) = \sum_{n=0}^{+\infty} h(n)\, e^{-j(\frac{2\pi l n}{N})} \qquad (3.6)$$

and its finite approximation

$$\tilde{H}(l) = \sum_{n=0}^{N-1} h(n)\, e^{-j(\frac{2\pi l n}{N})} \qquad (3.7)$$

The next step is to calculate the frequeccy response of the original system.

C. STRUCTURE OF THE PROGRAM

The algorithms introduced in the last chapter have been tested in the identification of a discrete time system. In particular we look at the problem of identifying the frequency response of the system by fitting a finite impulse response system to the input-output data.

The software developed consists of two programs: a main program which produces the input output data used in identification, and a subroutine which implements the Recursive Least Squares identification shown in Chapter II.

The effectiveness of the estimation is assessed on the basis of two criteria: the error between actual output of the system and predicted output, and the error between the frequency response of the system and the frequency response of its estimate.

Since we try to fit a finite impulse response model to a recursive system (with impulse response of infinite length), we cannot pretend to estimate the frequency response at all frequencies in an effective way. Therefore, we input test signals at finite number of frequencies and we look at the estimated spectrum at these frequencies only.

As it will be seen in the sequel, the results are satisfactory even in the presence of added disturbances in the measurements. The recursive use of blocks of data has the effect of smoothing the effect of disturbances in the estimates.

D. SIMULATION RESULTS

In this thesis, we try inputs with different frequencies to test the identification algorithm. Also, we test the behavior when the input is the combination of several different frequencies to examine the effectiveness of the technique. In the next paragraph, we discuss some results obtained by running the program.

1. Original System

Figure 3.1 shows the magnitude and phase of frequency response of the original system. It shows the low pass nature of the system.

2. Single Frequency Input at Arbitrary Frequency without Noise

Figure 3.2 and 3.3 are in the same group in which we tried a constant input with magnitude 10. As predicted, figure 3.2 portrays the error EPS between the output and predicted output which decreases and approaches zero. Furthermore, in figure 3.3, the frequency response of the estimated model at the input frequency is shown to be identical to the one of the original system.

In a second set of experiments, we tried several sinusoidal inputs with different frequencies. First of all, we used an input signal with a digital frequency $\pi/2$ and obtain the results shown in figures 3.4 and 3.5. And then, we tried another input with frequency $\pi/8$ and get the results shown in figures 3.6 and 3.7. In like manner, the error approaches to zero quickly and the frequency response of the estimated model at the input frequency is the same as the one of the orignal system.

3. Multiple Frequency Inputs without Noise

In a third set of experiments, we tried the combination of four frequency inputs at arbitrary frequencies to obtain the results shown in figures 3.8, 3.9, and 3.10. The output in the time domain is shown in figure 3.8. In this case, the error between true and predicted outputs approaches zero quickly and the frequency responses at the input frequencies are still identical.

4. Single Frequency Input with Noise

In this section we show results of similar experiments with random measurement noise. The noise considered is Gaussian white zero mean, i.e., independently and indentically distributed. Its standard deviation is set at about one third of the output magnitude.

As before we first excite the system with a constant input, and we add measurement noise to the output of the system. The results are shown in figures 3.11, 3.12, and 3.13. From figure 3.12, even though we find that the error does not decay to zero due to the presence of noise, the frequency response of the estimated model is sufficiently close to the one of the original system.

In a second set of experiments, we tried sinusoidal inputs again with noisy measurements. There are four inputs with different digital frequencies ($11\pi/16$, $\pi/2$, $\pi/4$, and $\pi/16$) used and the results are shown in figures (3.14-3.22) respectively. Then, by inspecting the figures, the error still does not go to zero due to the presence of noise, but the estimated frequency response is sufficiently close to the one of the original system.

5. Multiple Frequency Inputs with Noise

In like manner, we input signals with multiple frequencies to test the alogrithm. In this case, we used four different groups of inputs. The first one is constant and sinusoidal (frequency $\pi/2$) input. The 2nd one is a constant and sinusoidal (frequency $\pi/4$) input. The 3rd and 4th are sinusoidal with two frequencies $(\pi/2, \pi/4)$ and $(\pi/4, \pi/8)$ respectively. The results are shown in figures 3.23 to 3.31. Then, inspection of these figures reveals that the error in each case is bounded and the frequency responses of the estimated model are sufficiently close to the one of the original system.

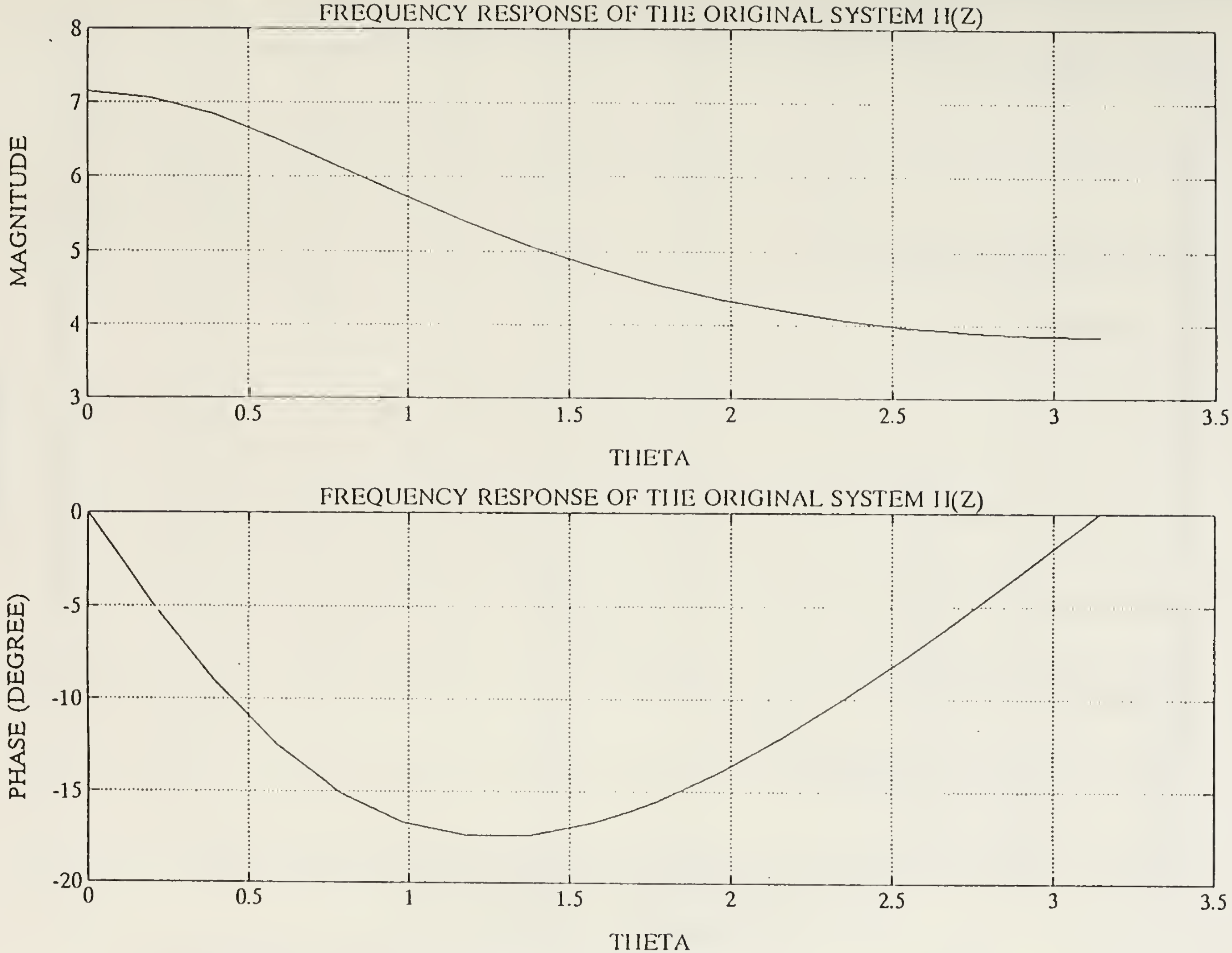

Figure 3.1 Frequency Response of the Original System

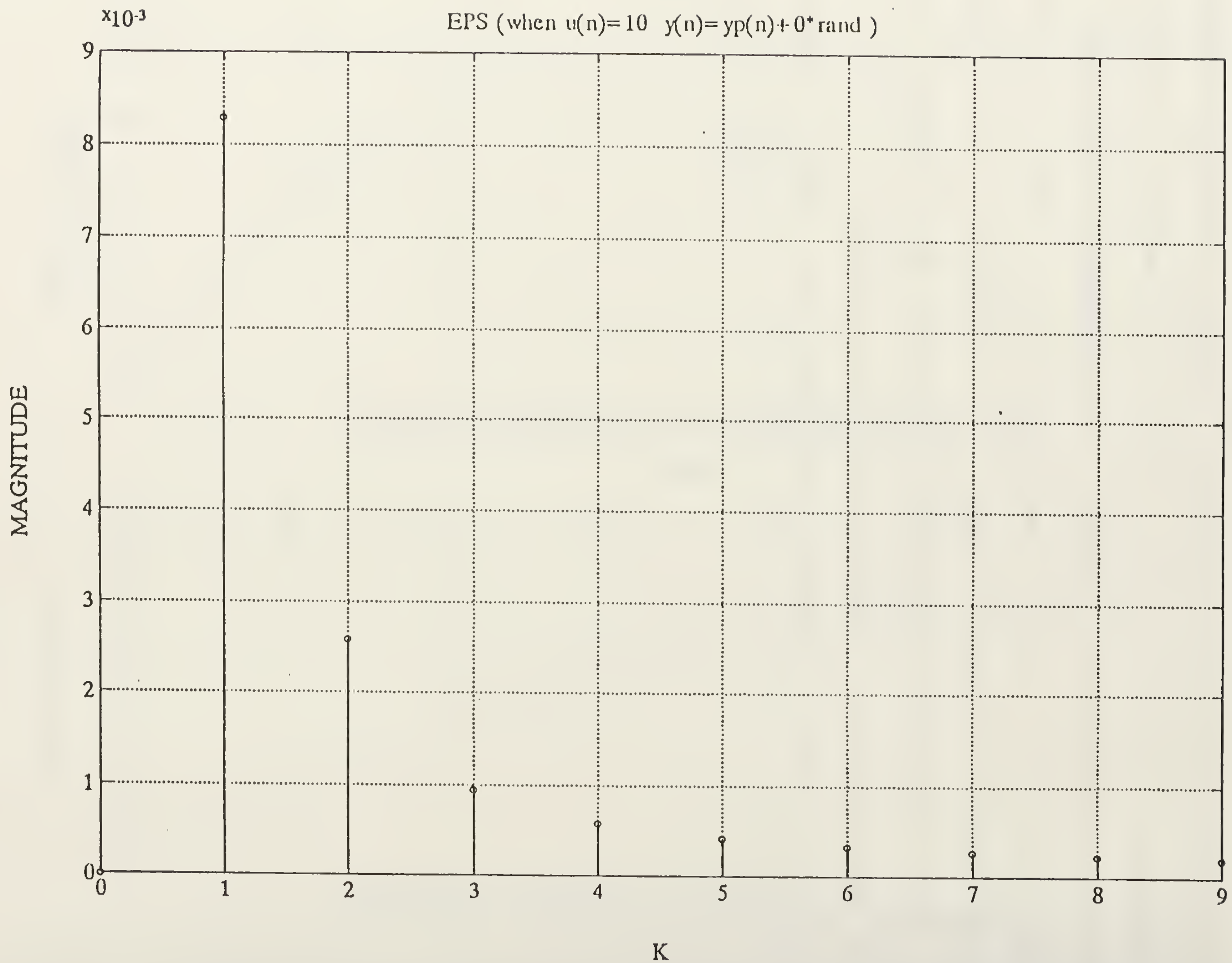

Figure 3.2 Error Between Original and Predicted Output

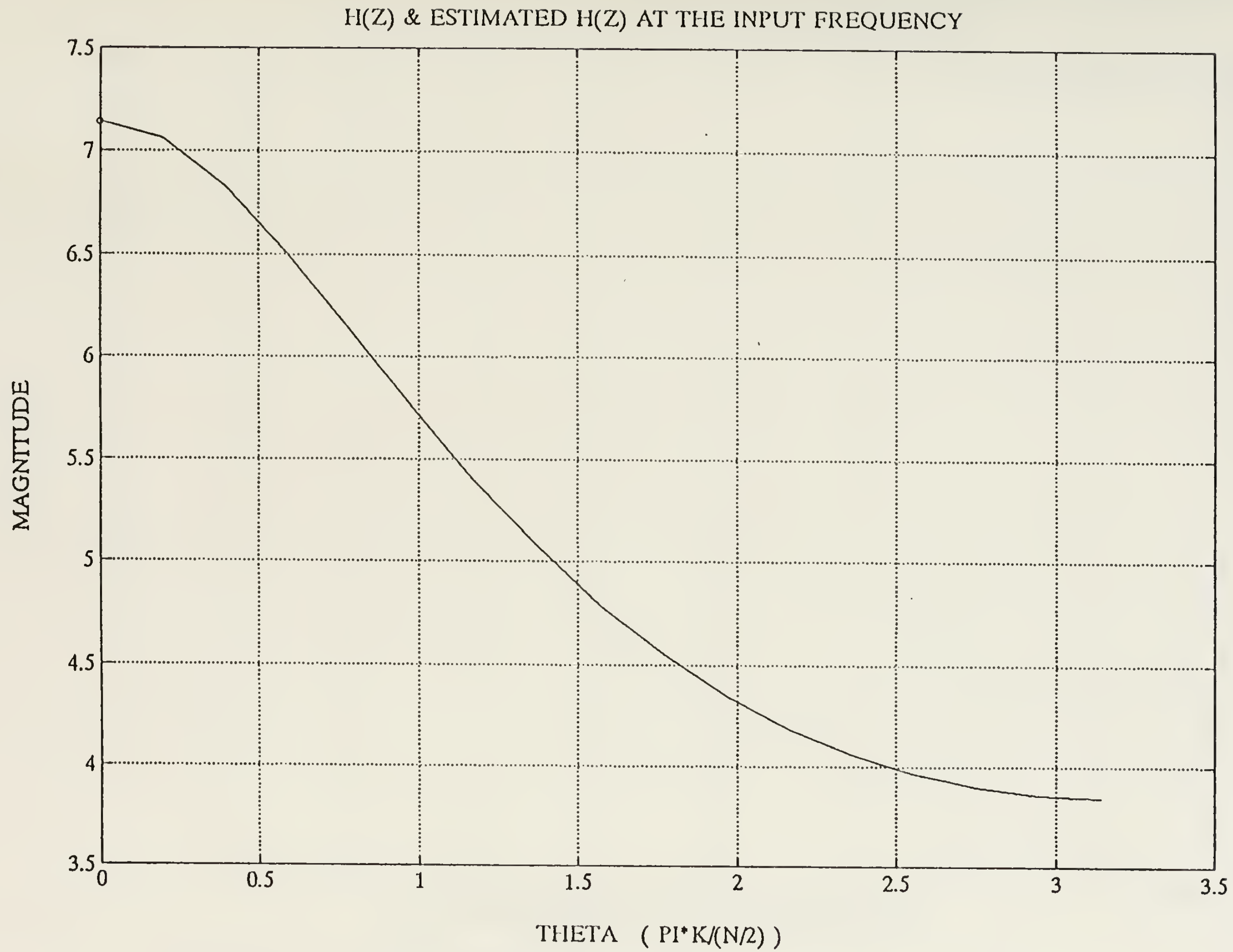

Figure 3.3 The Original and Estimated Frequency Response

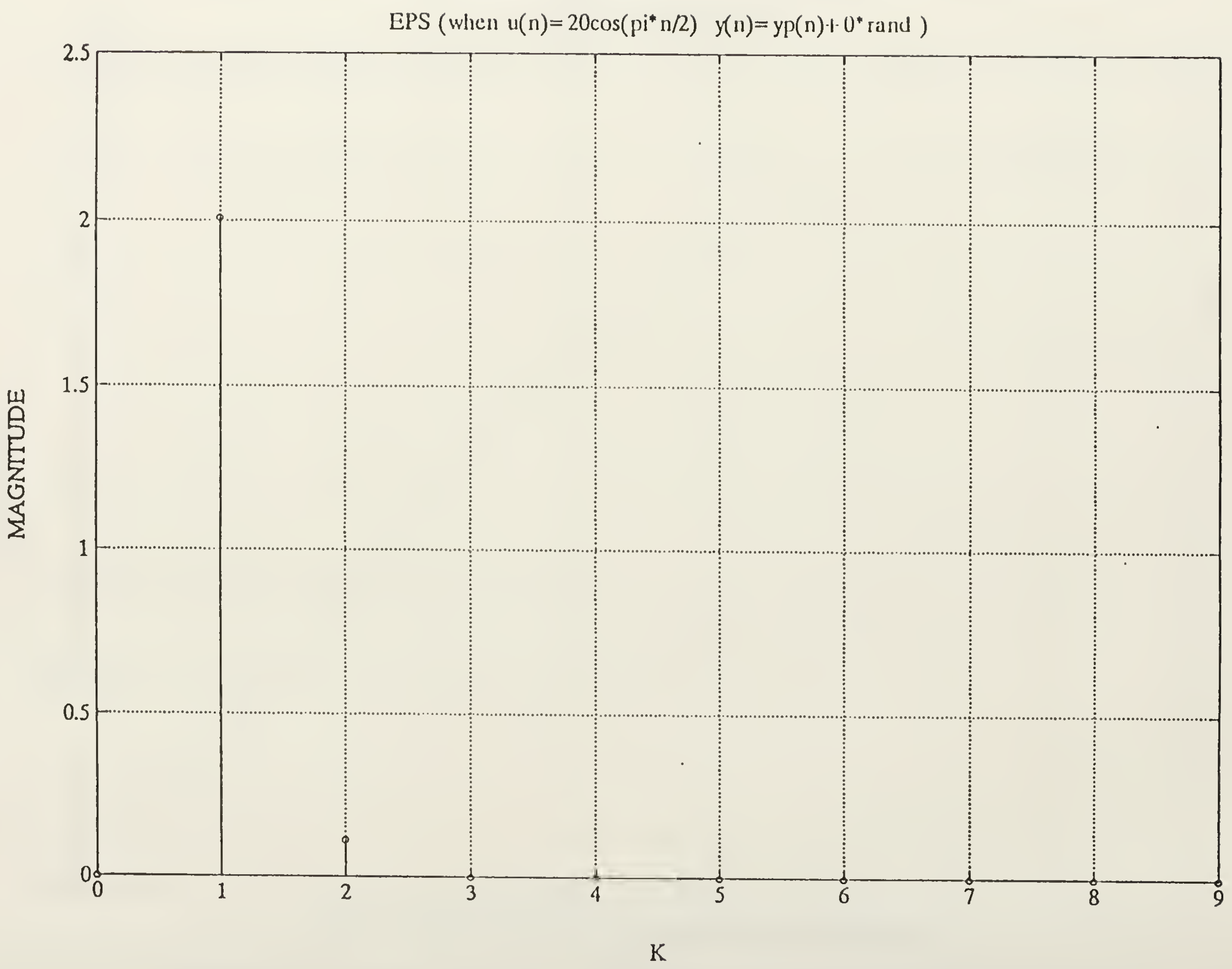

Figure 3.4 Error Between Original and Predicted Output

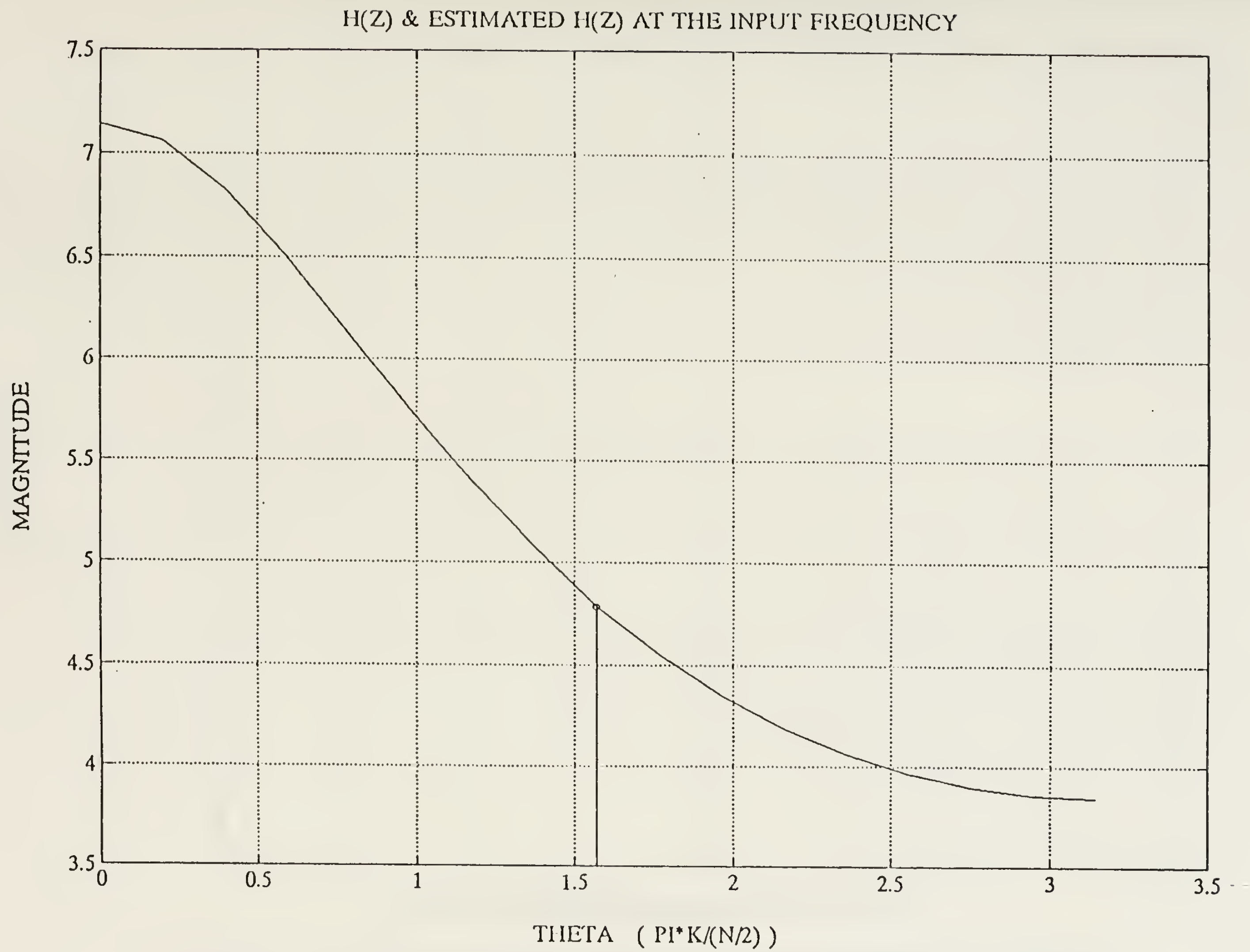

Figure 3.5 The Original and Estimated Frequency Response

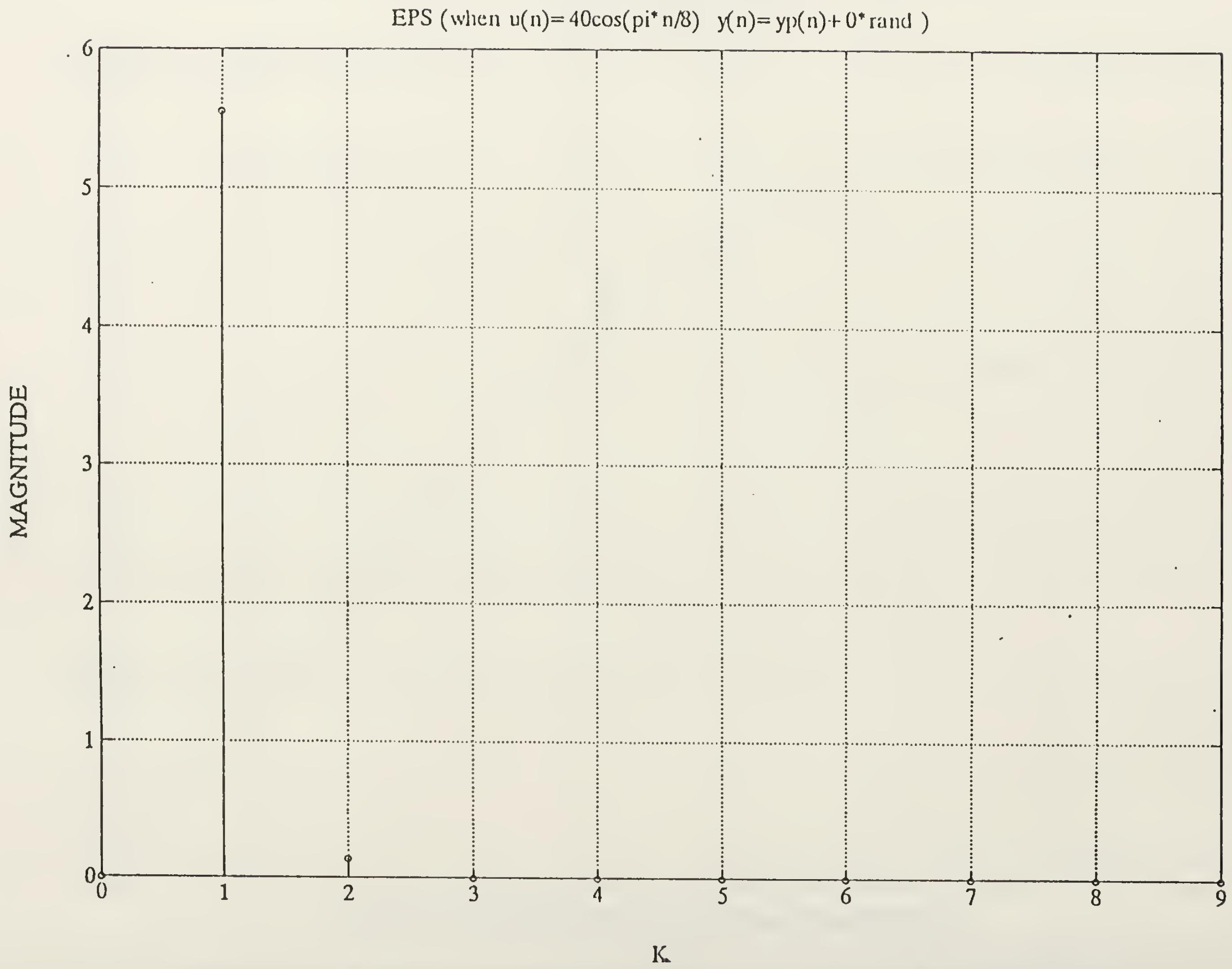

Figure 3.6 Error Between Original and Predicted Output

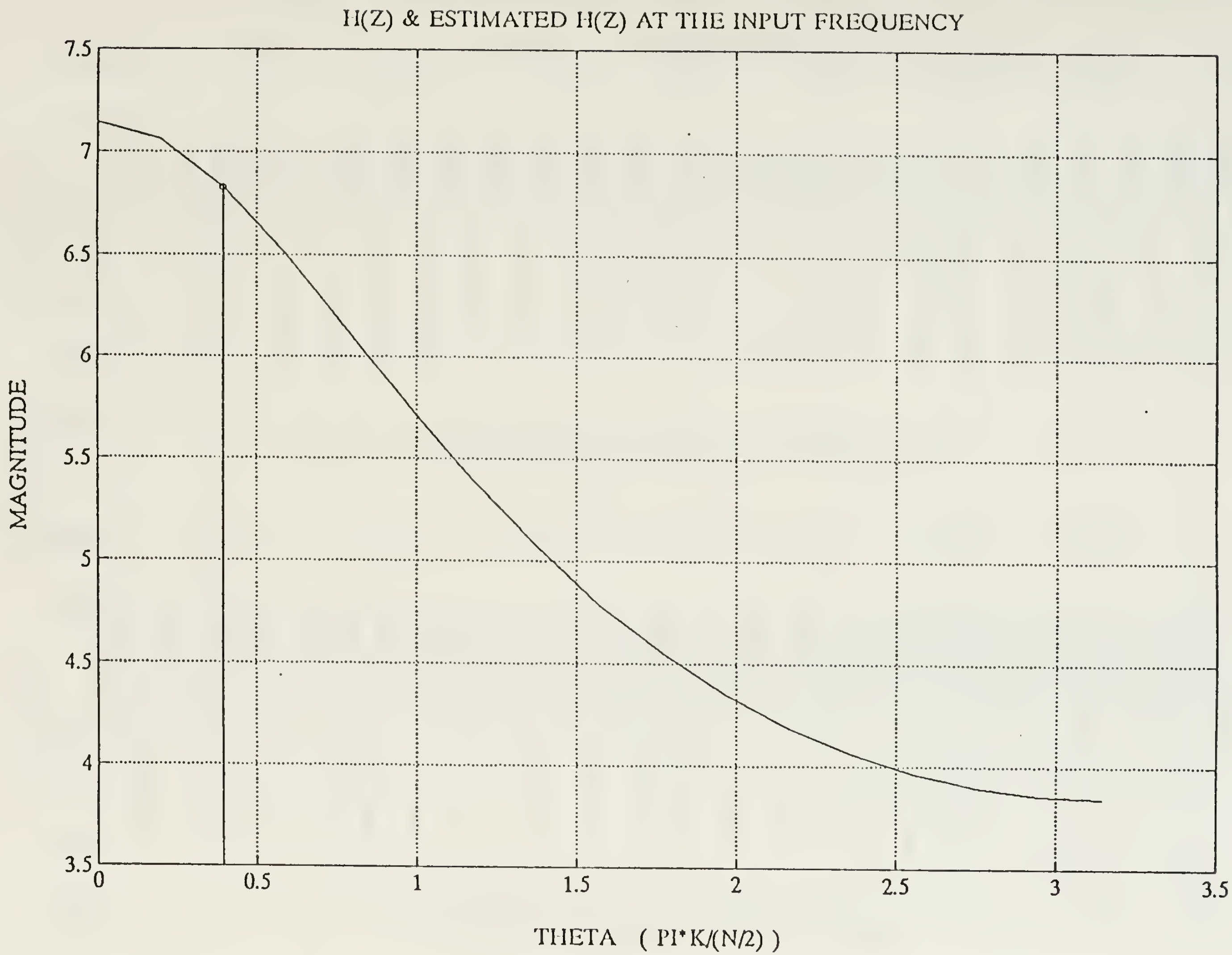

Figure 3.7 The Original and Estimated Frequency Response

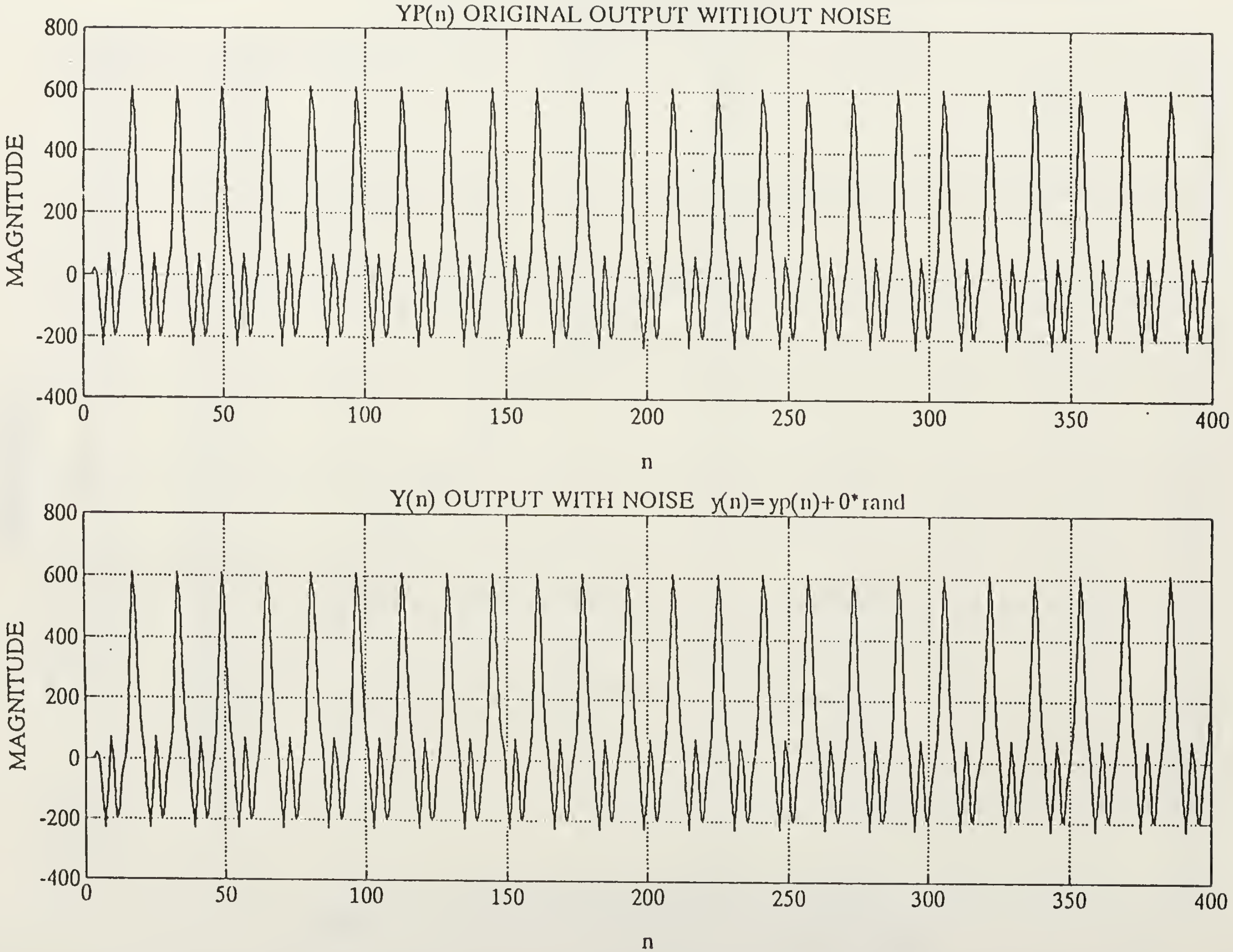

Figure 3.8 The Original Output without Noise

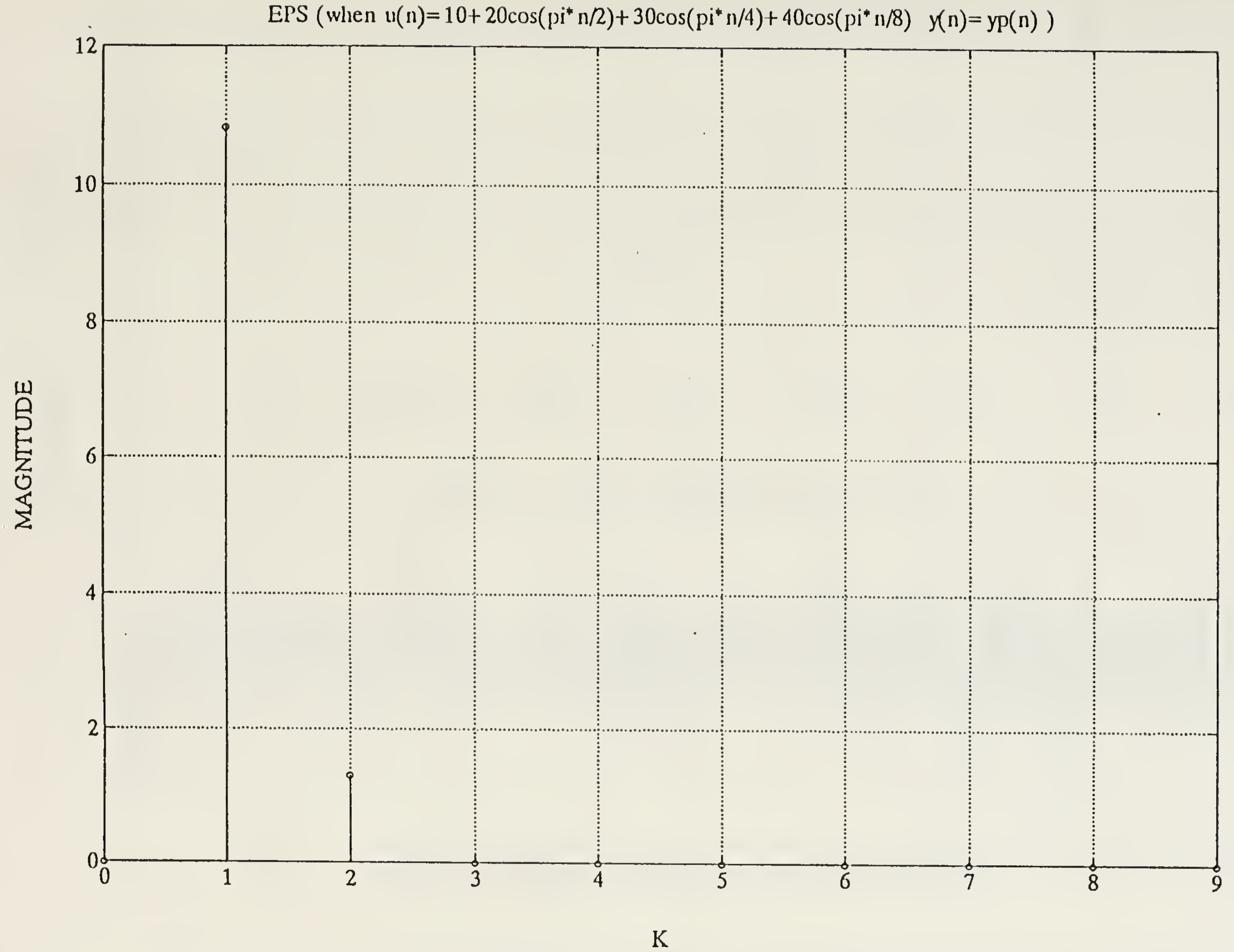

Figure 3.9 Error Between Original and Predicted Output

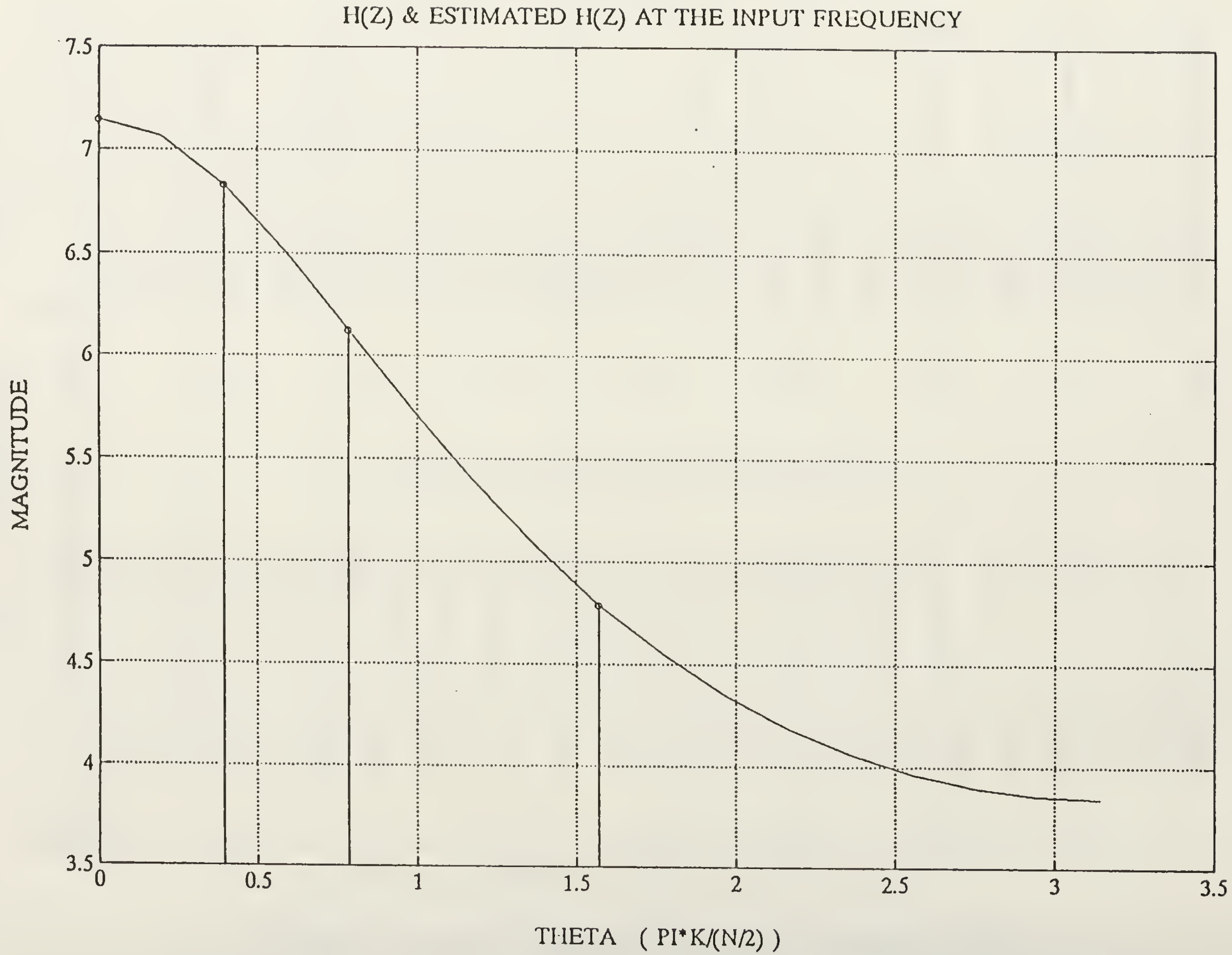

Figure 3.10 The Original and Estimated Frequency Response

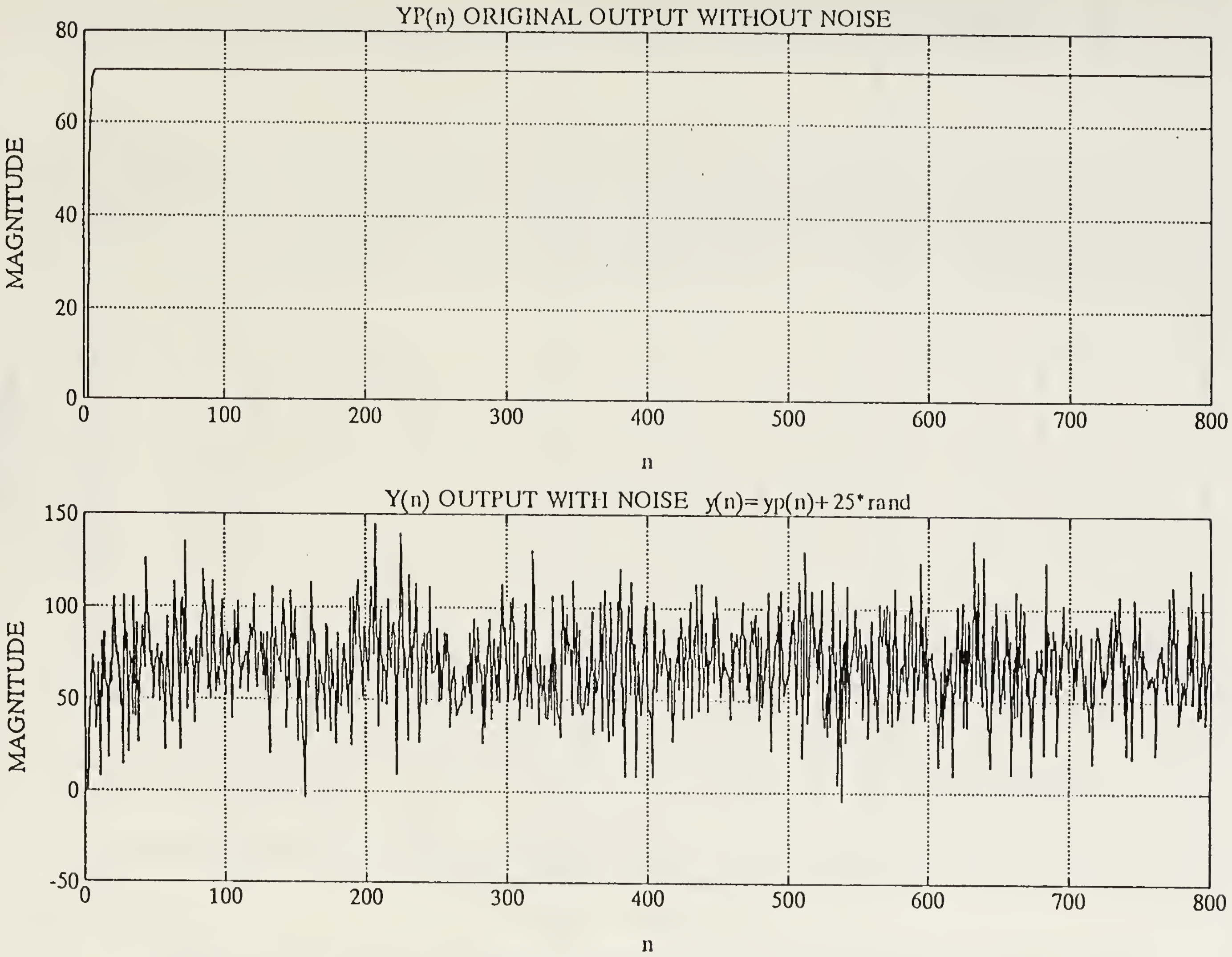

Figure 3.11 The Original Output with Noise and without Noise

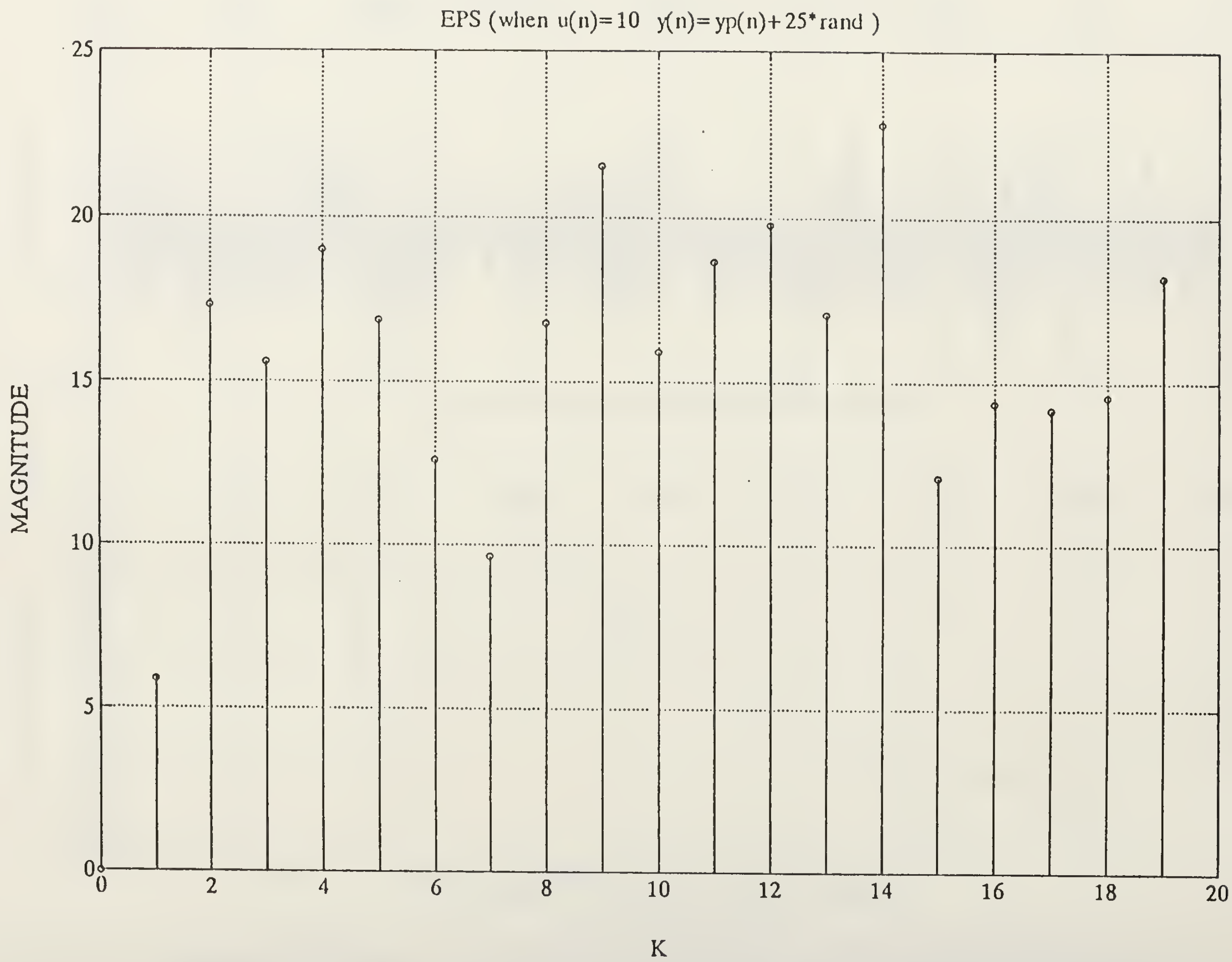

Figure 3.12 Error Between Original and Predicted Output

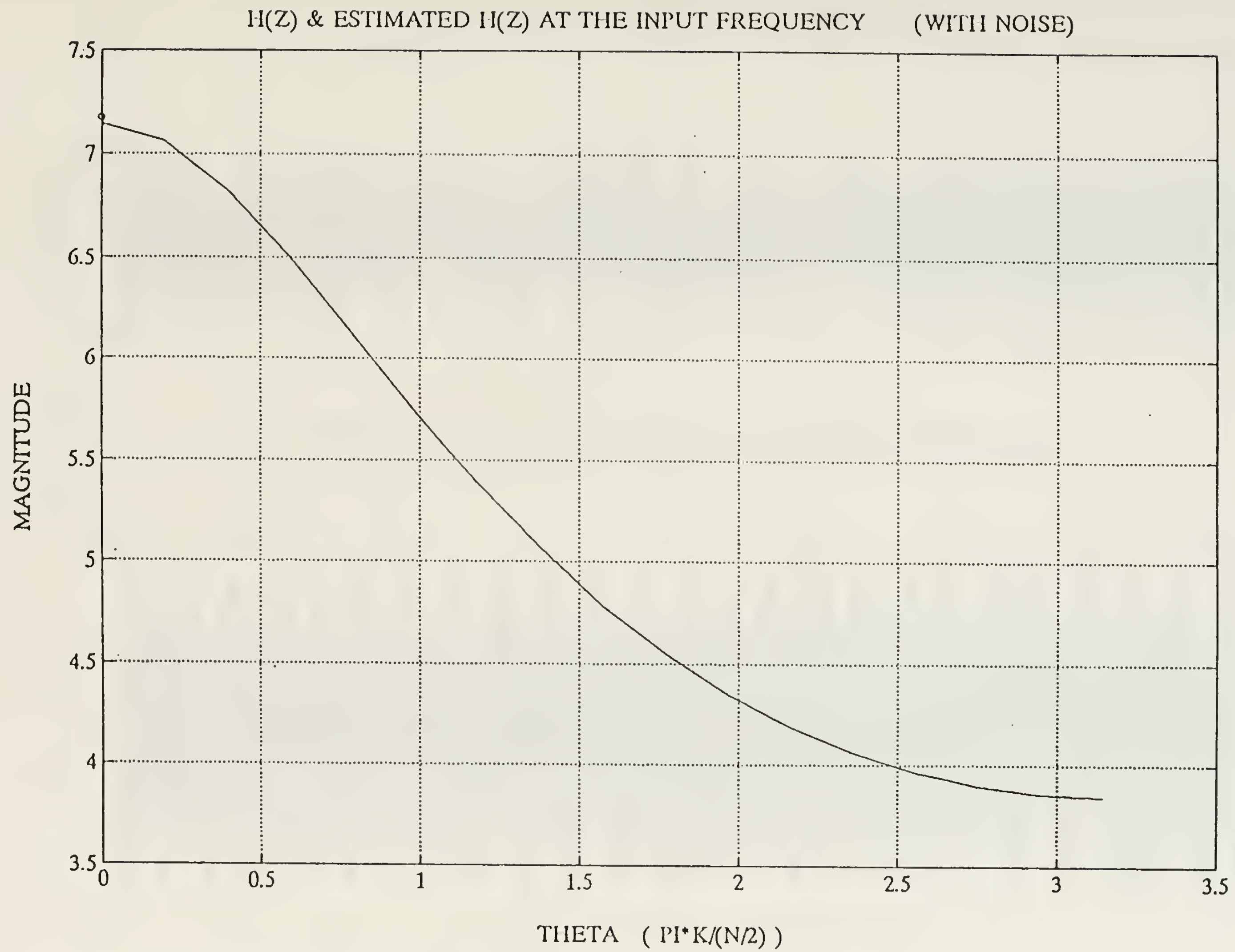

Figure 3.13 The Original and Estimated Frequency Response

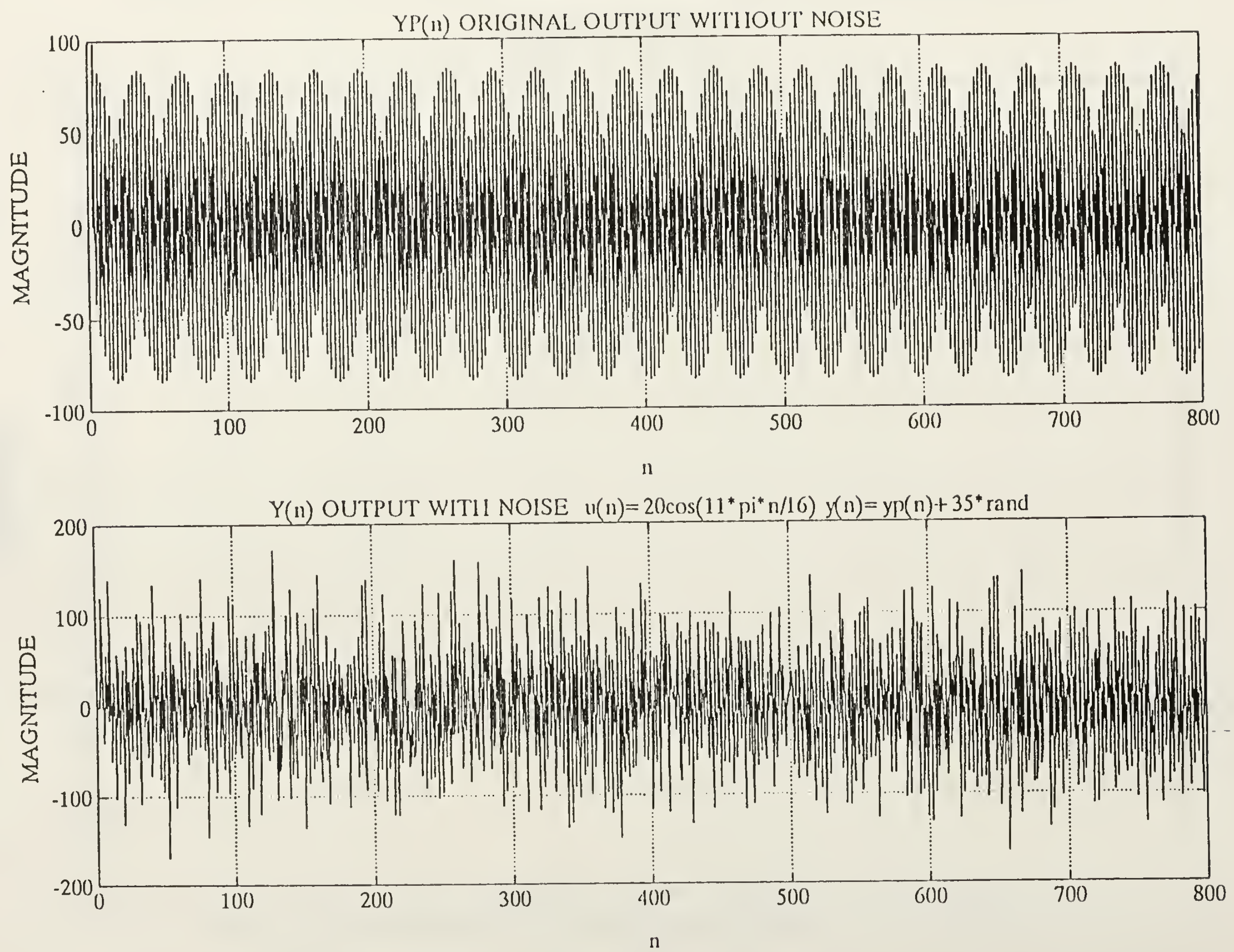

Figure 3.14 The Original Output with Noise and without Noise

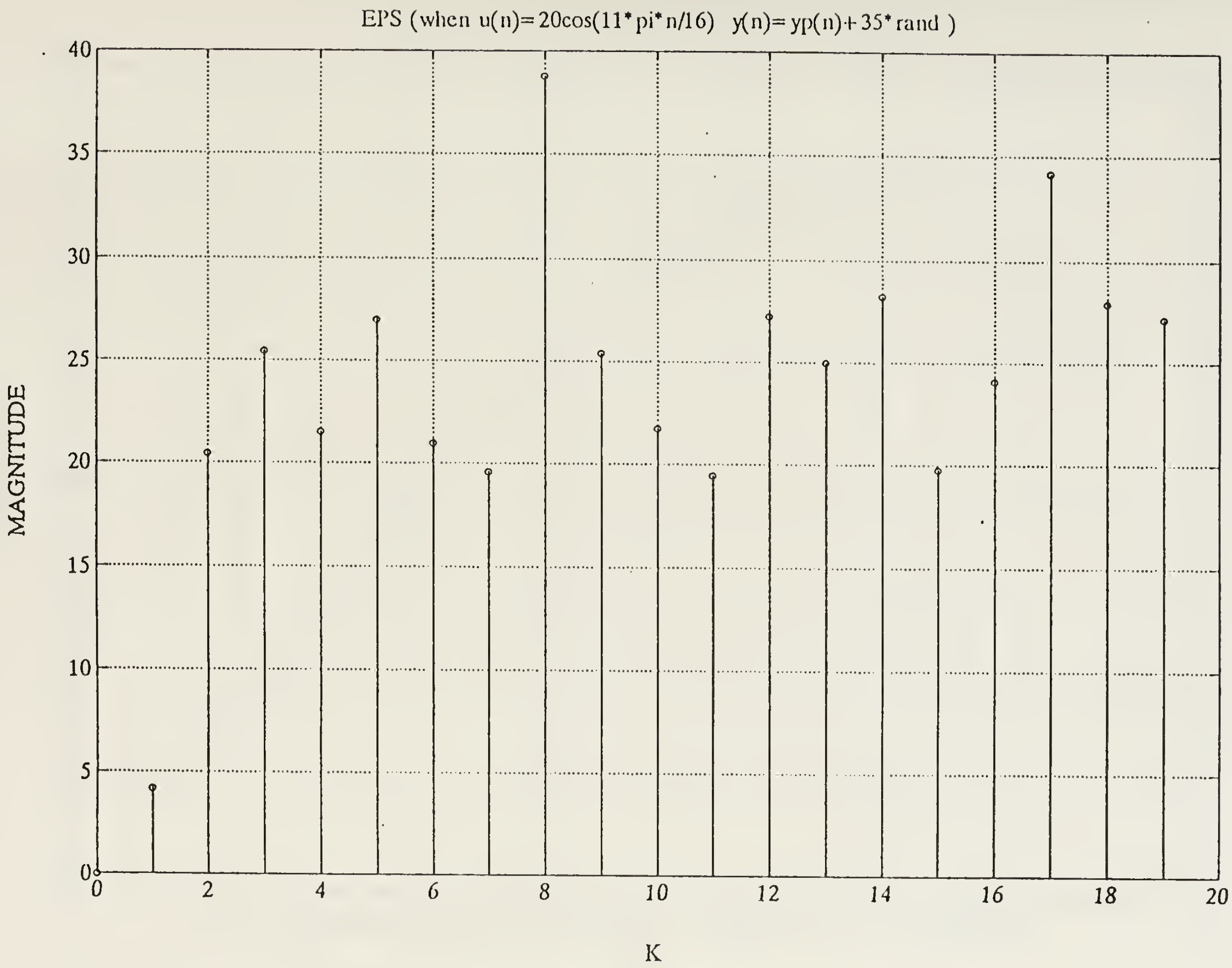

Figure 3.15 Error Between Original and Predicted Output

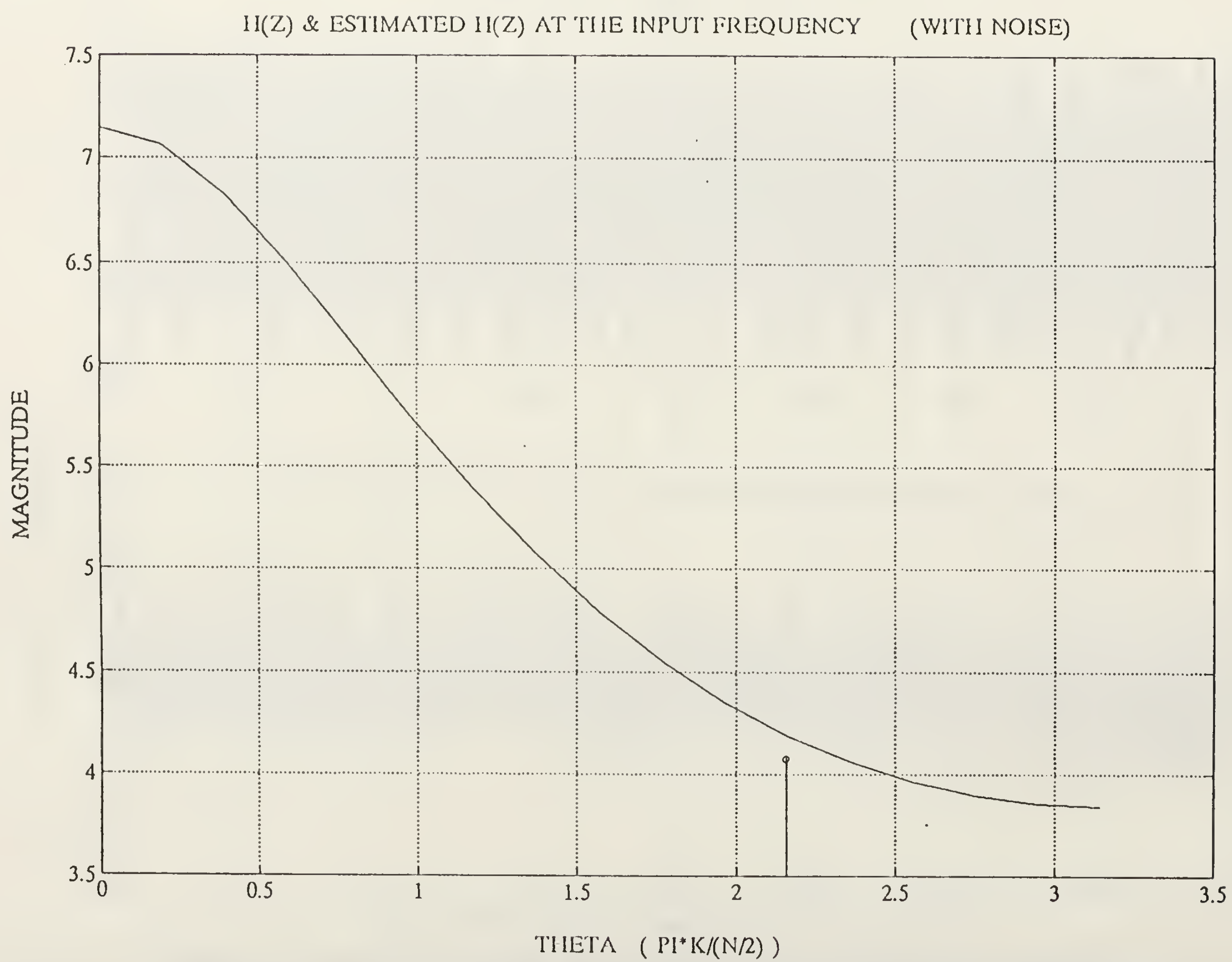

Figure 3.16 The Original and Estimated Frequency Response

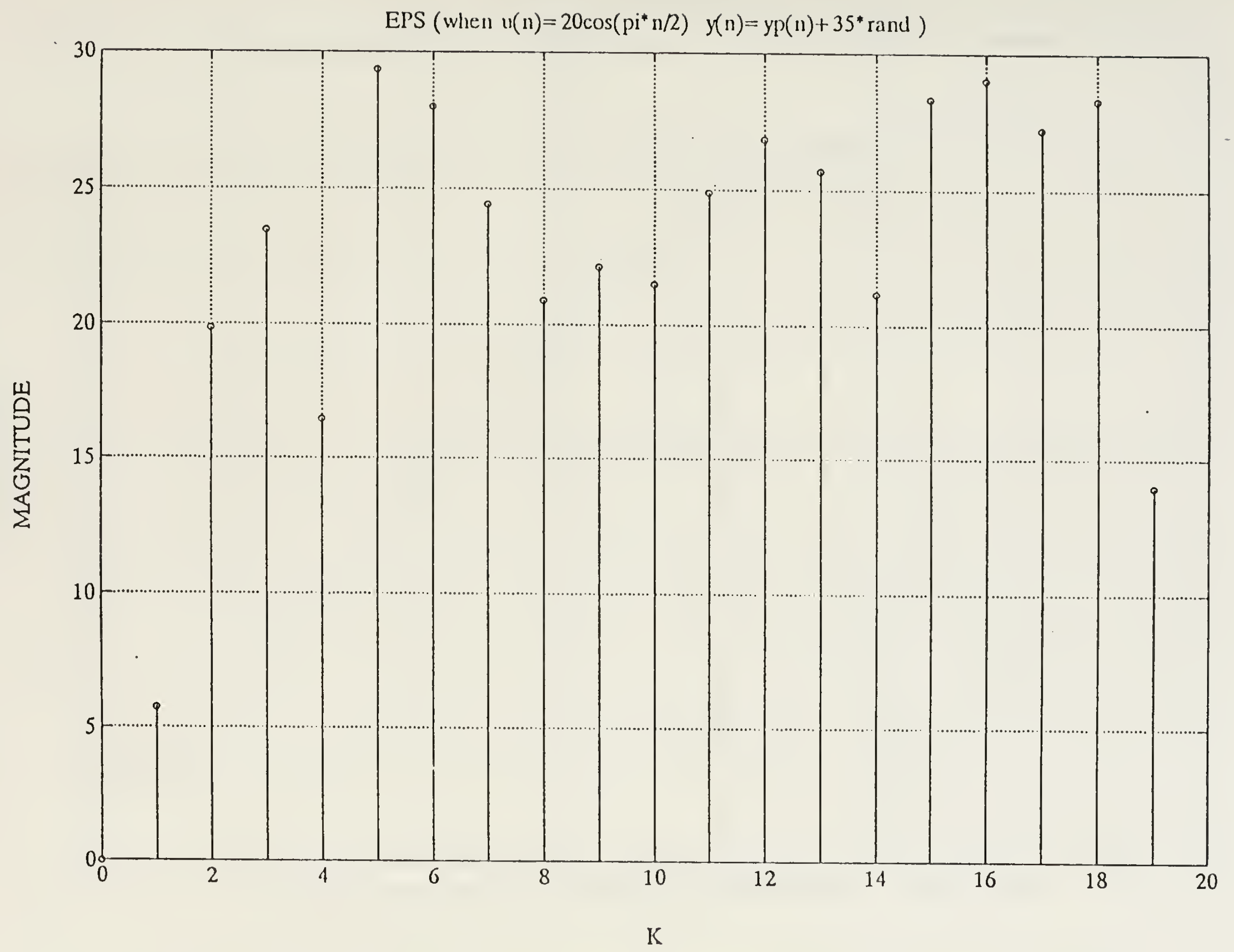

Figure 3.17 Error Between Original and Predicted Output

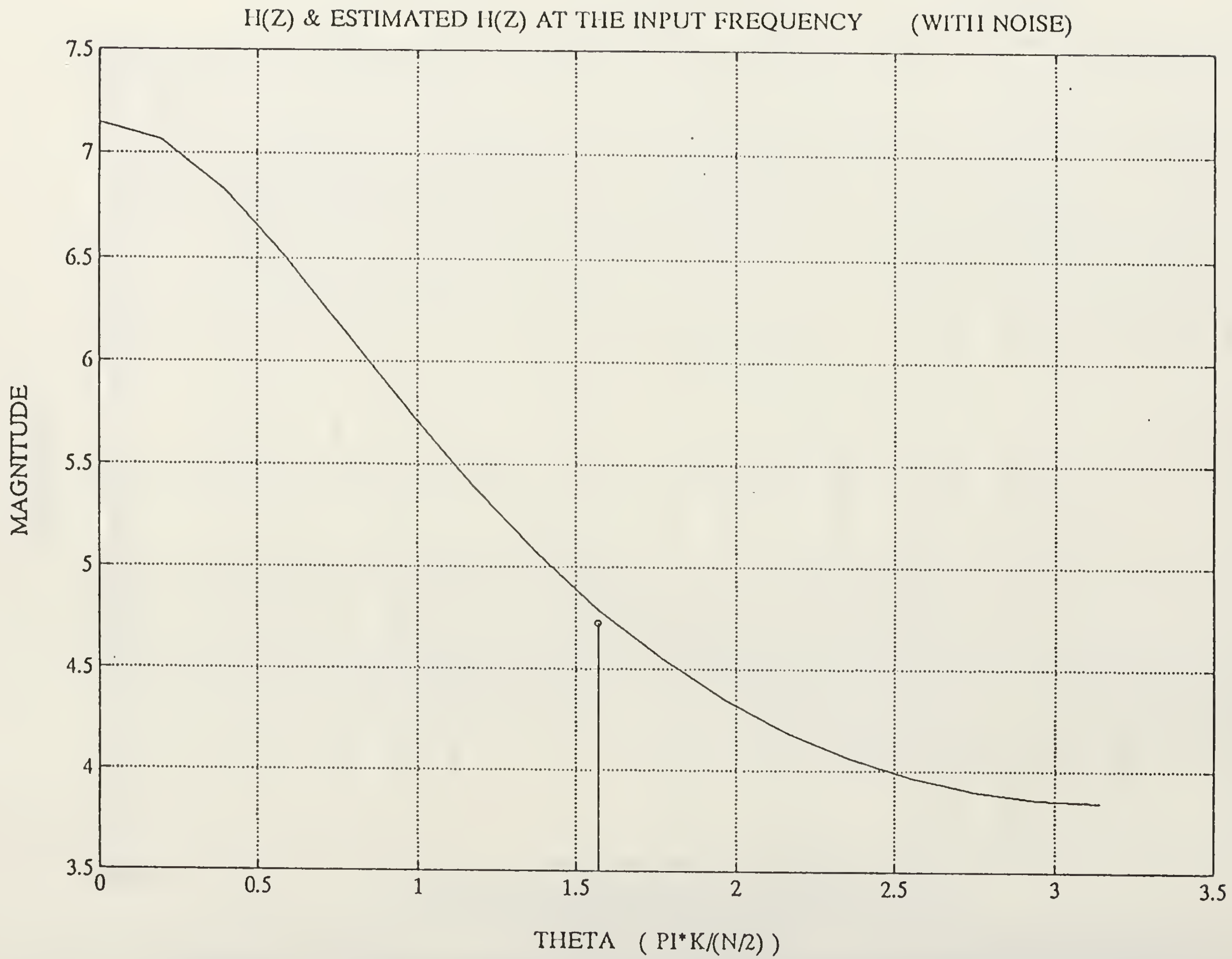

Figure 3.18 The Original and Estimated Frequency Response

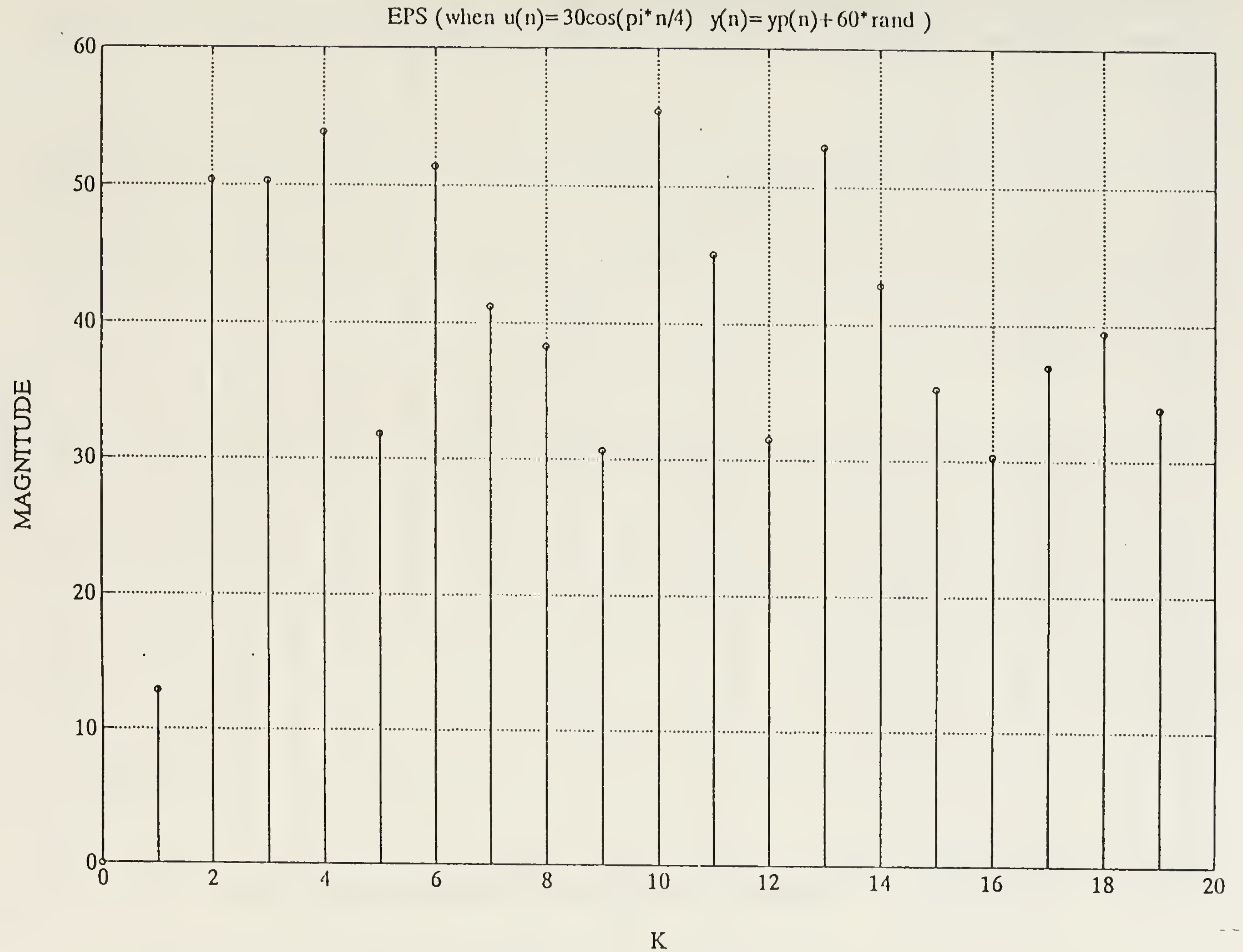

Figure 3.19 Error Between Original and Predicted Output

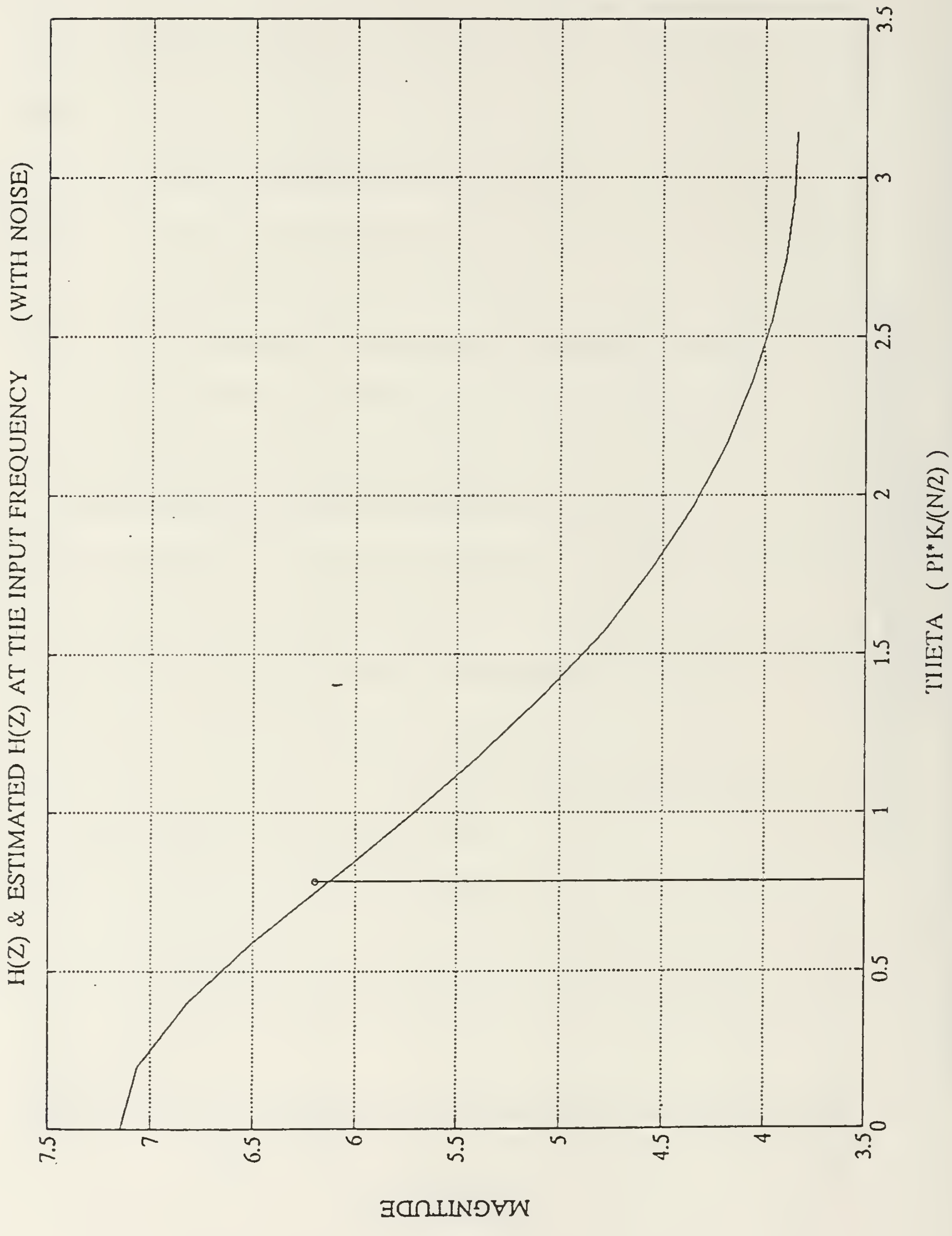

Figure 3.20 The Original and Estimated Frequency Response

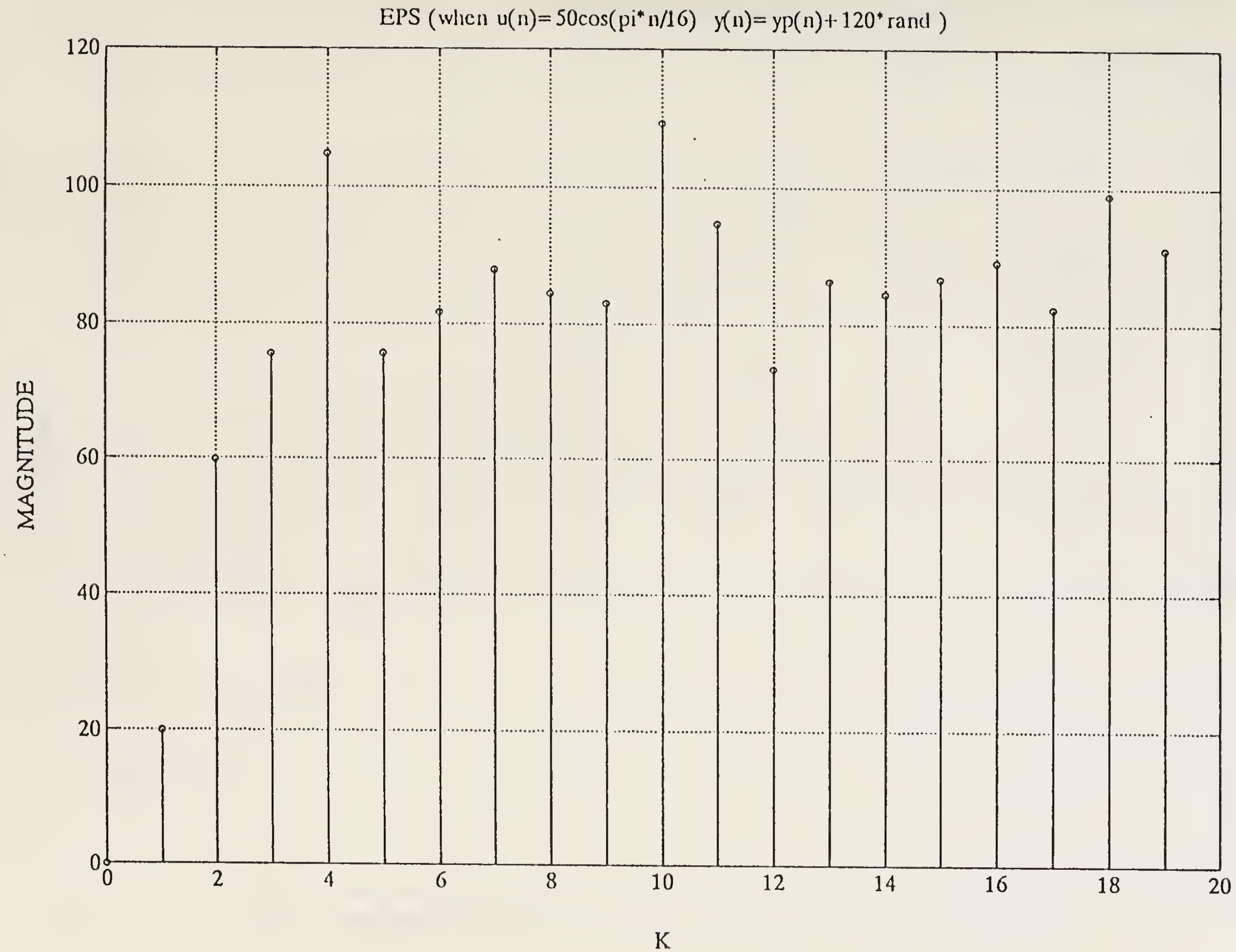

Figure 3.21 Error Between Original and Predicted Output

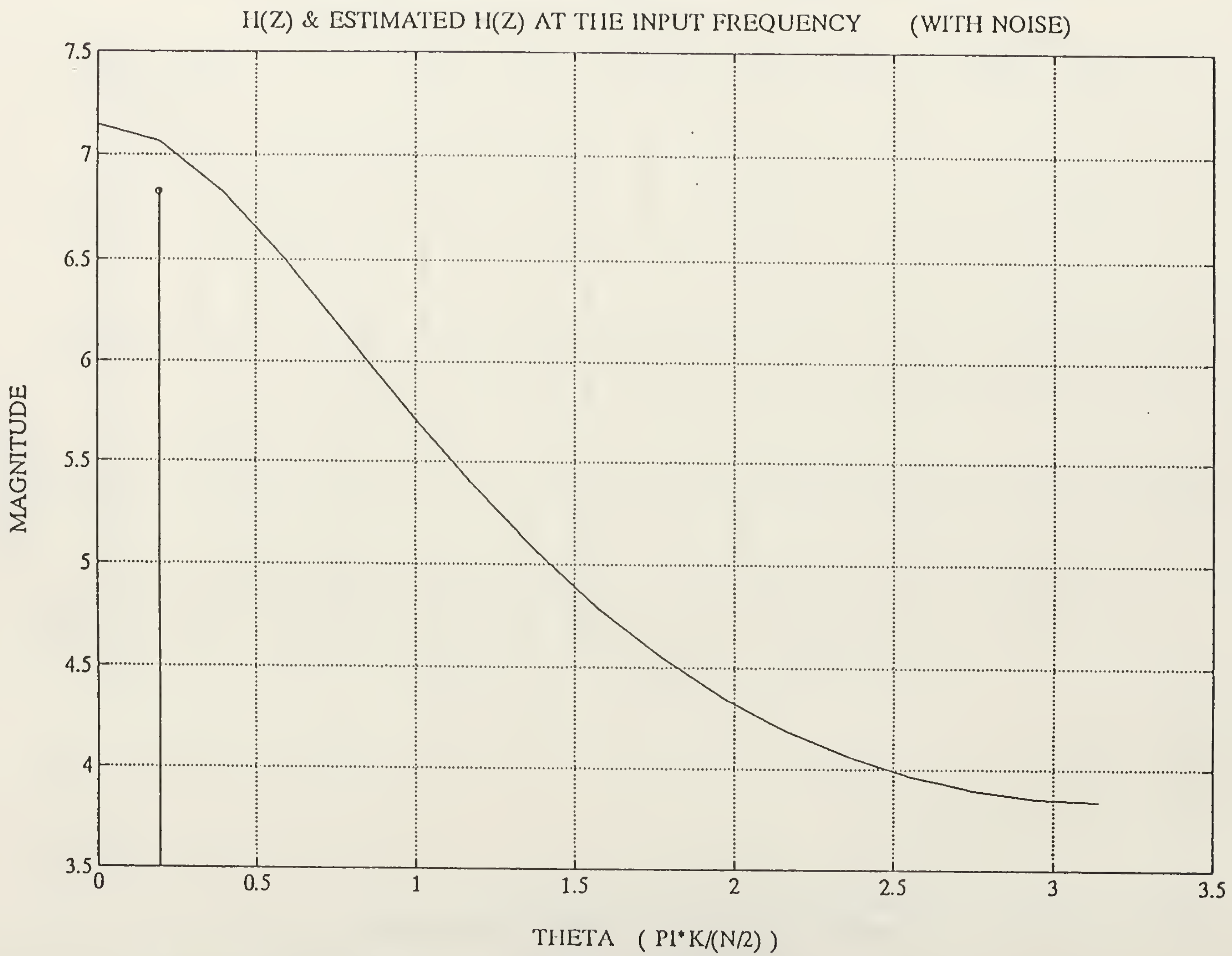

Figure 3.22 The Original and Estimated Frequency Response

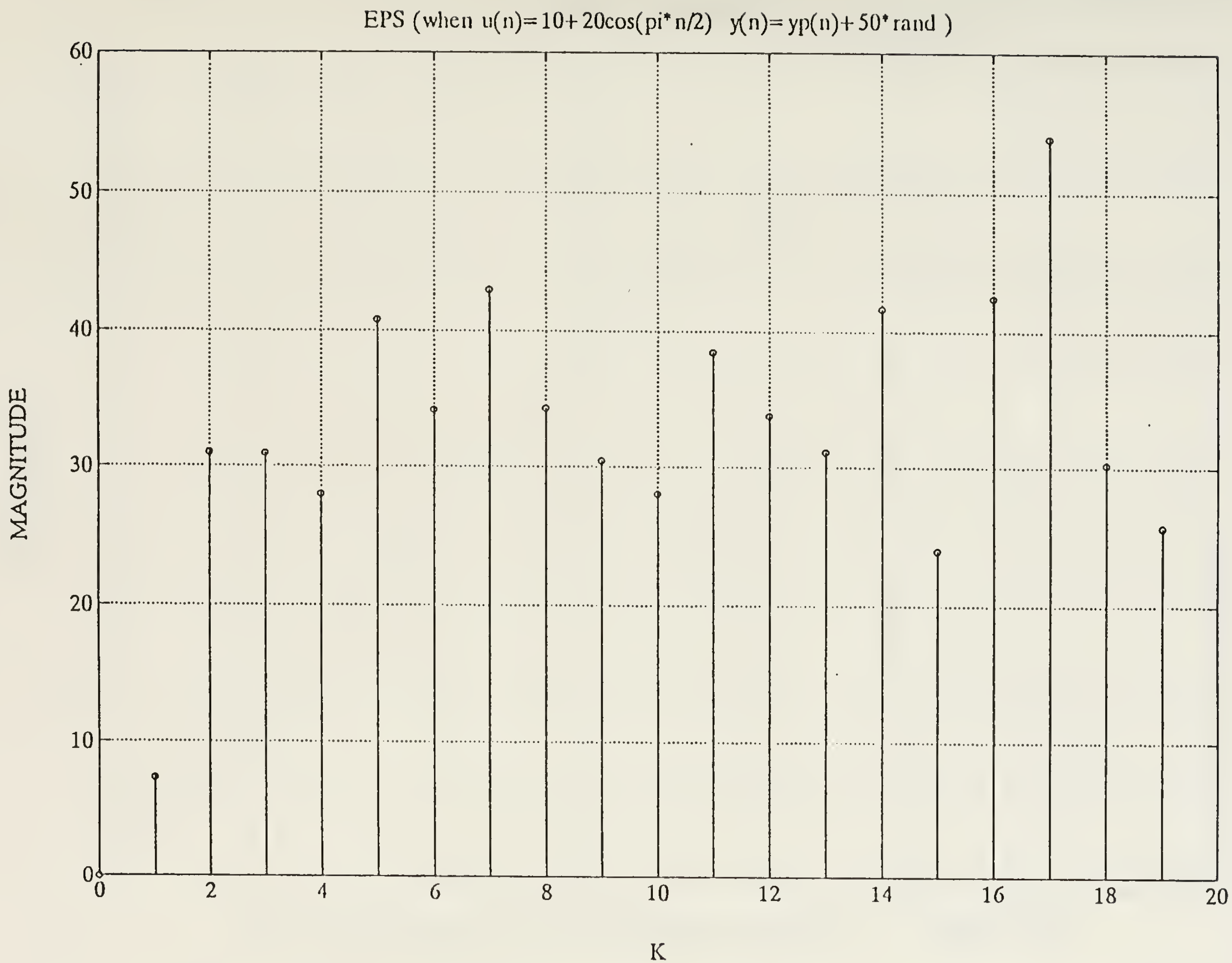

Figure 3.23 Error Between Original and Predicted Output

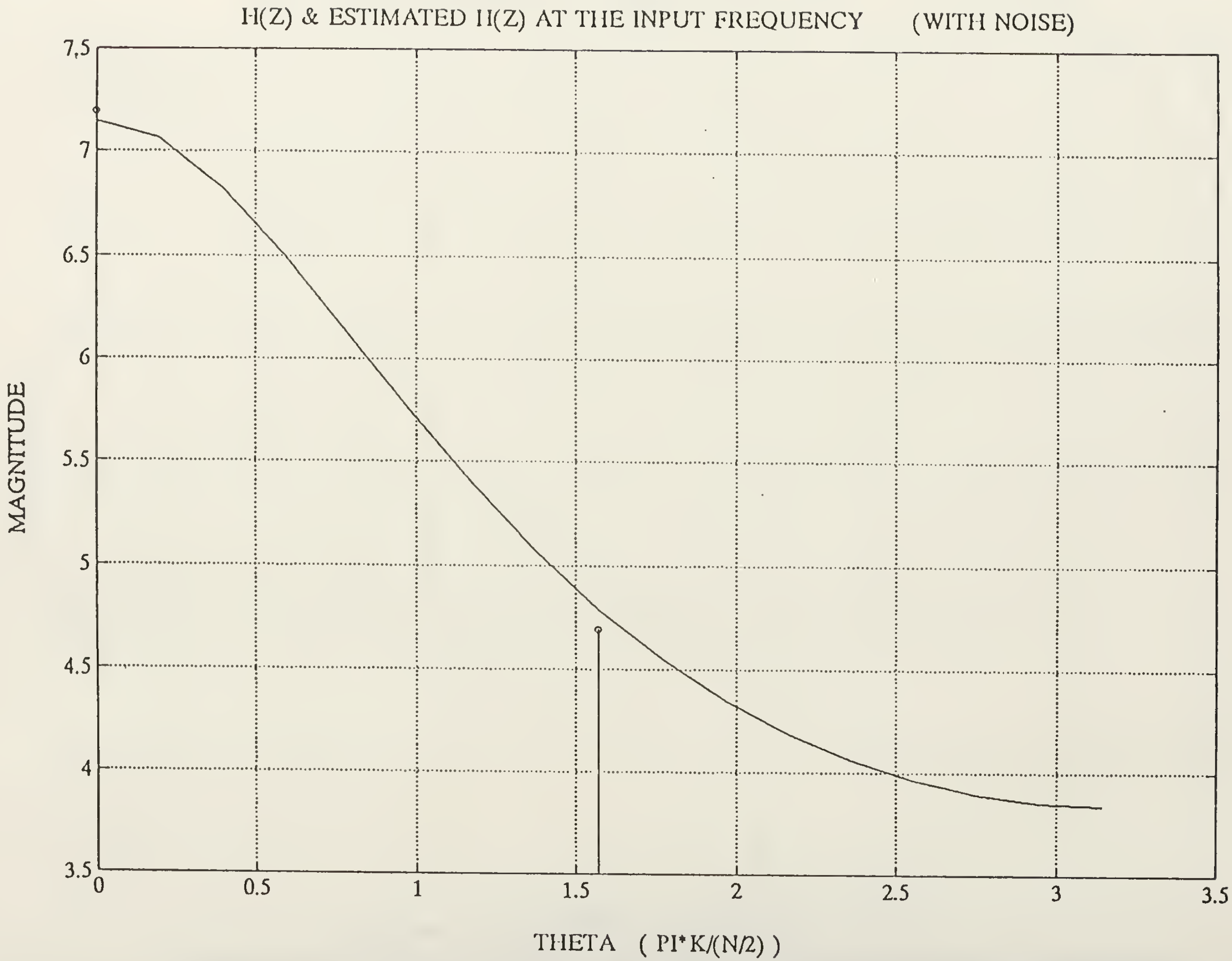

Figure 3.24 The Original and Estimated Frequency Response

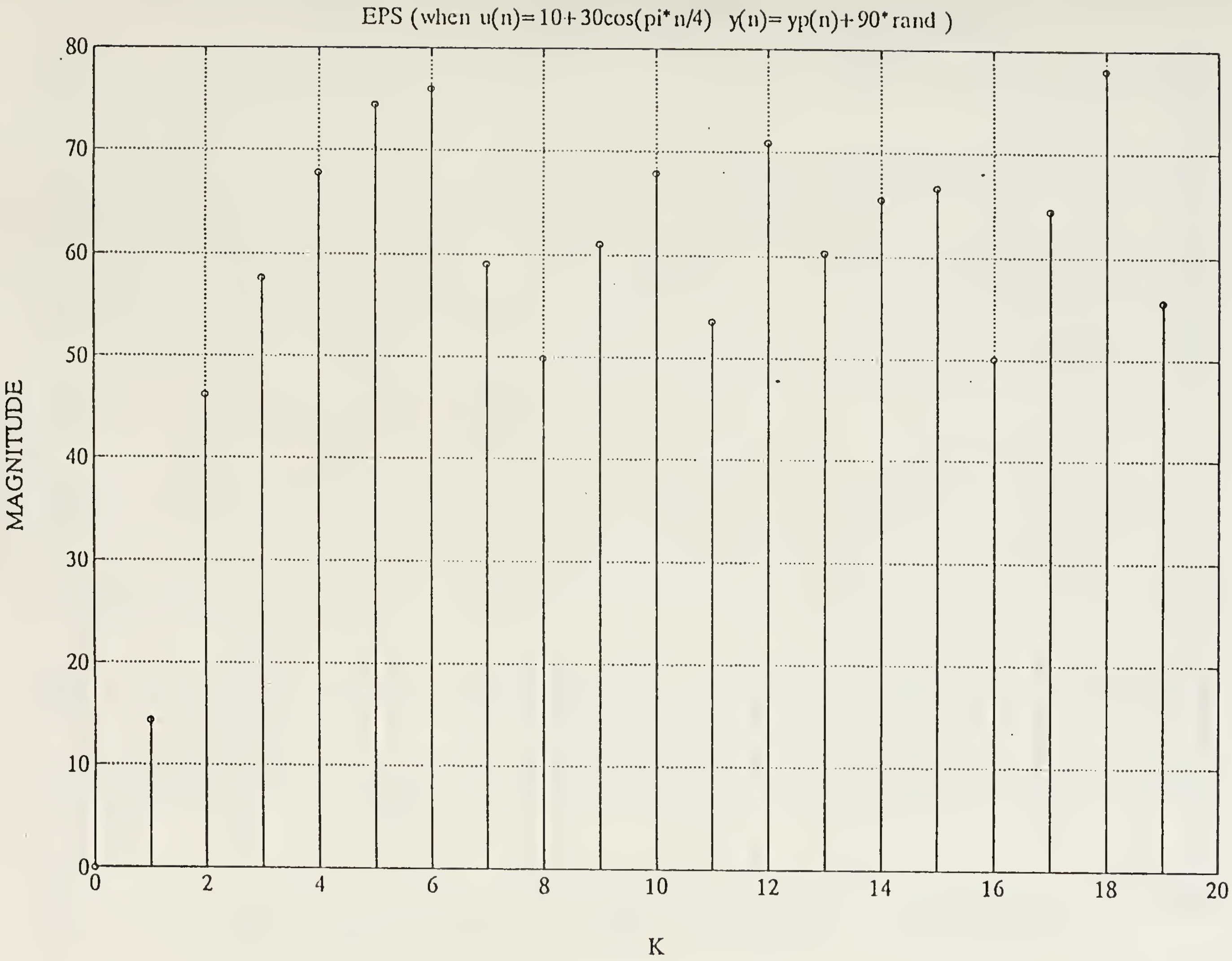

Figure 3.25 Error Between Original and Predicted Output

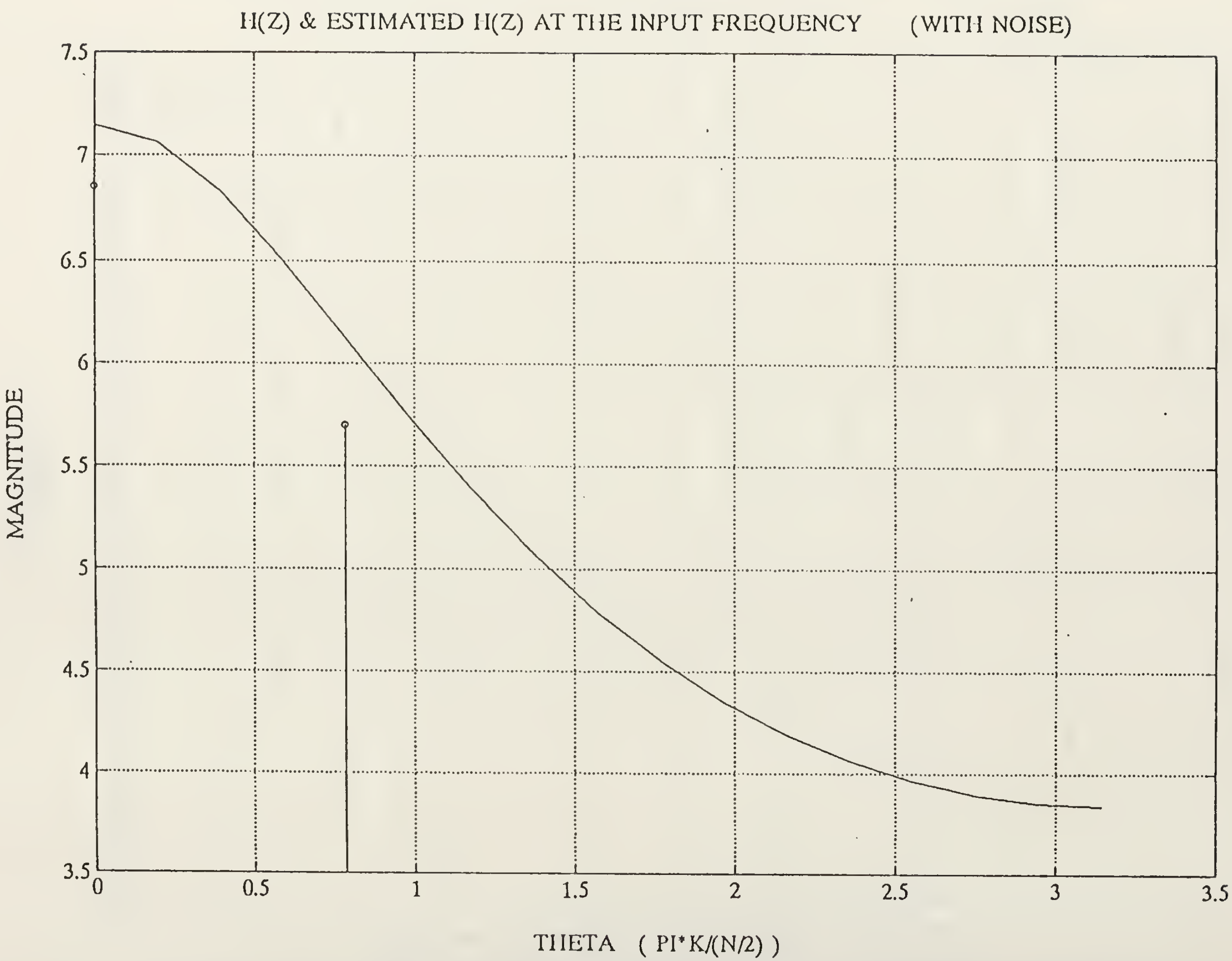

Figure 3.26 The Original and Estimated Frequency Response

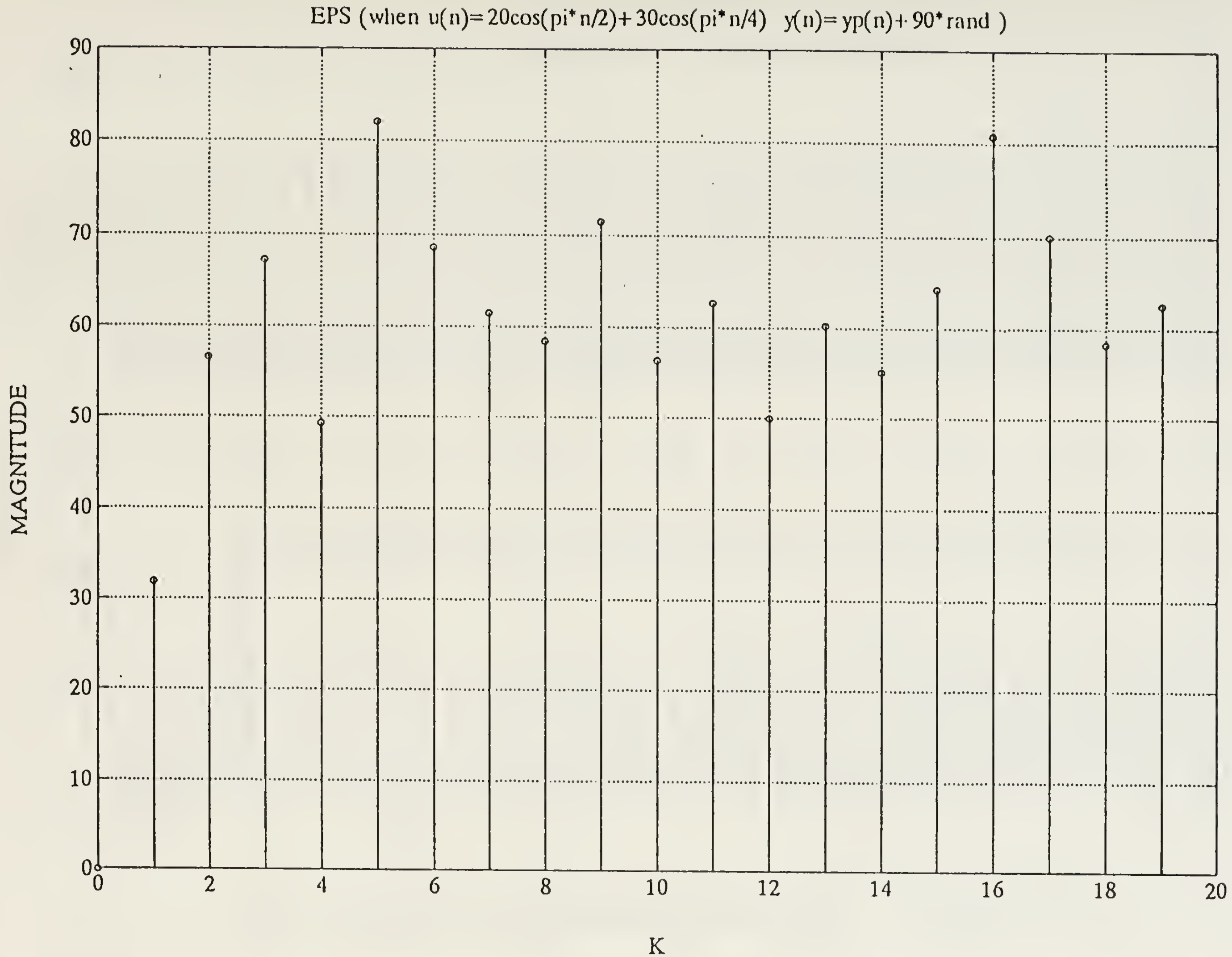

Figure 3.27 Error Between Original and Predicted Output

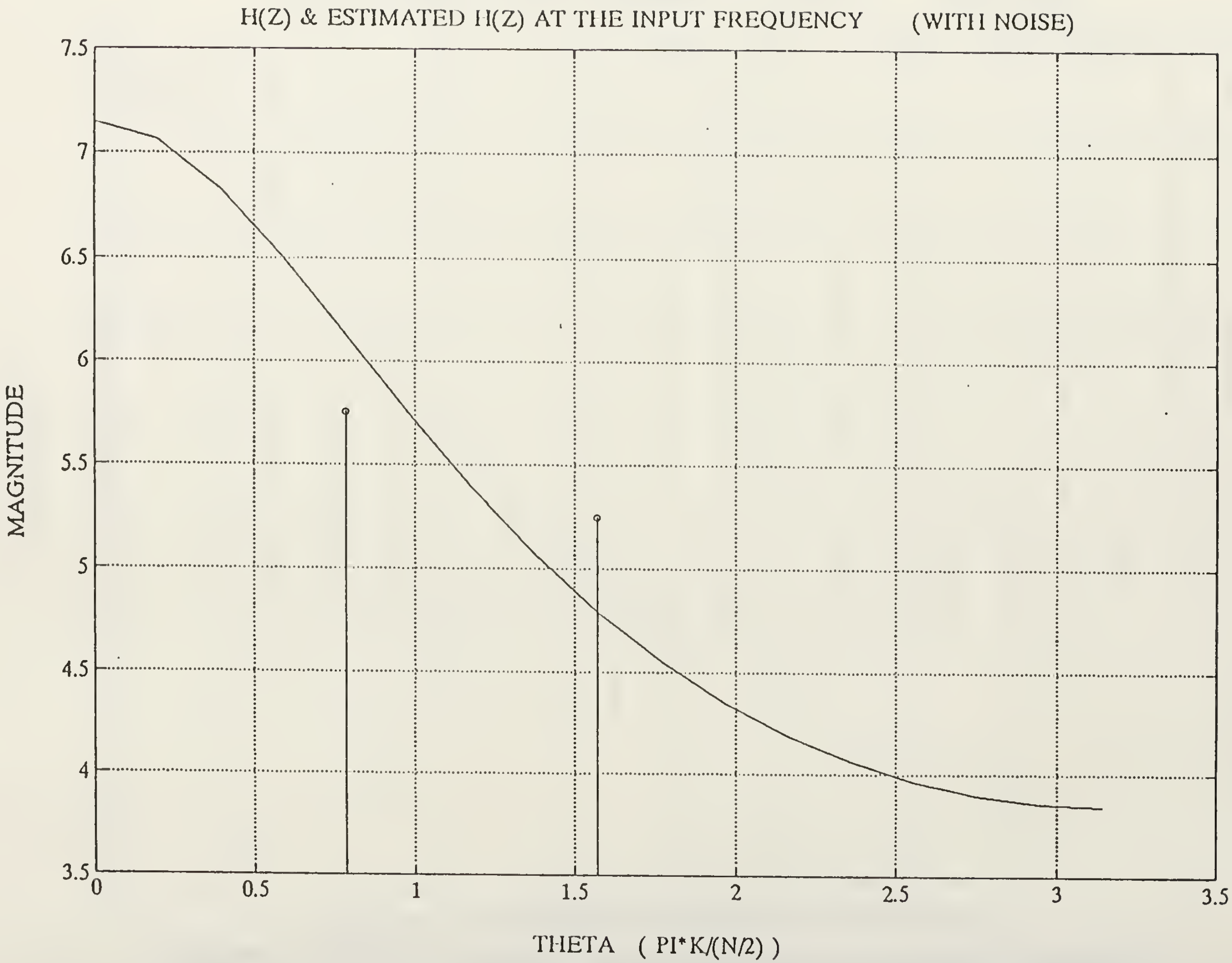

Figure 3.28 The Original and Estimated Frequency Response

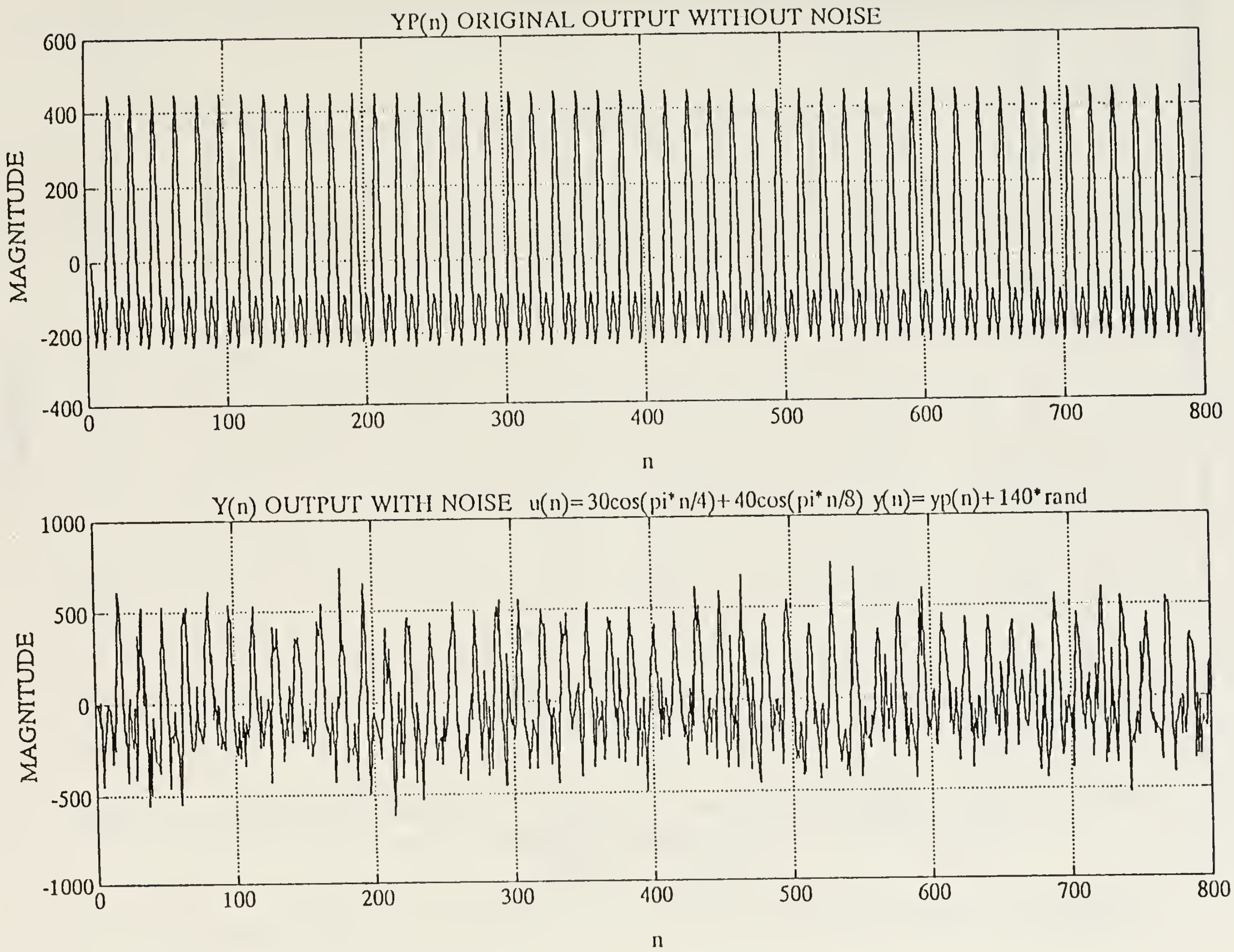

Figure 3.29 The Original Output with Noise and without Noise

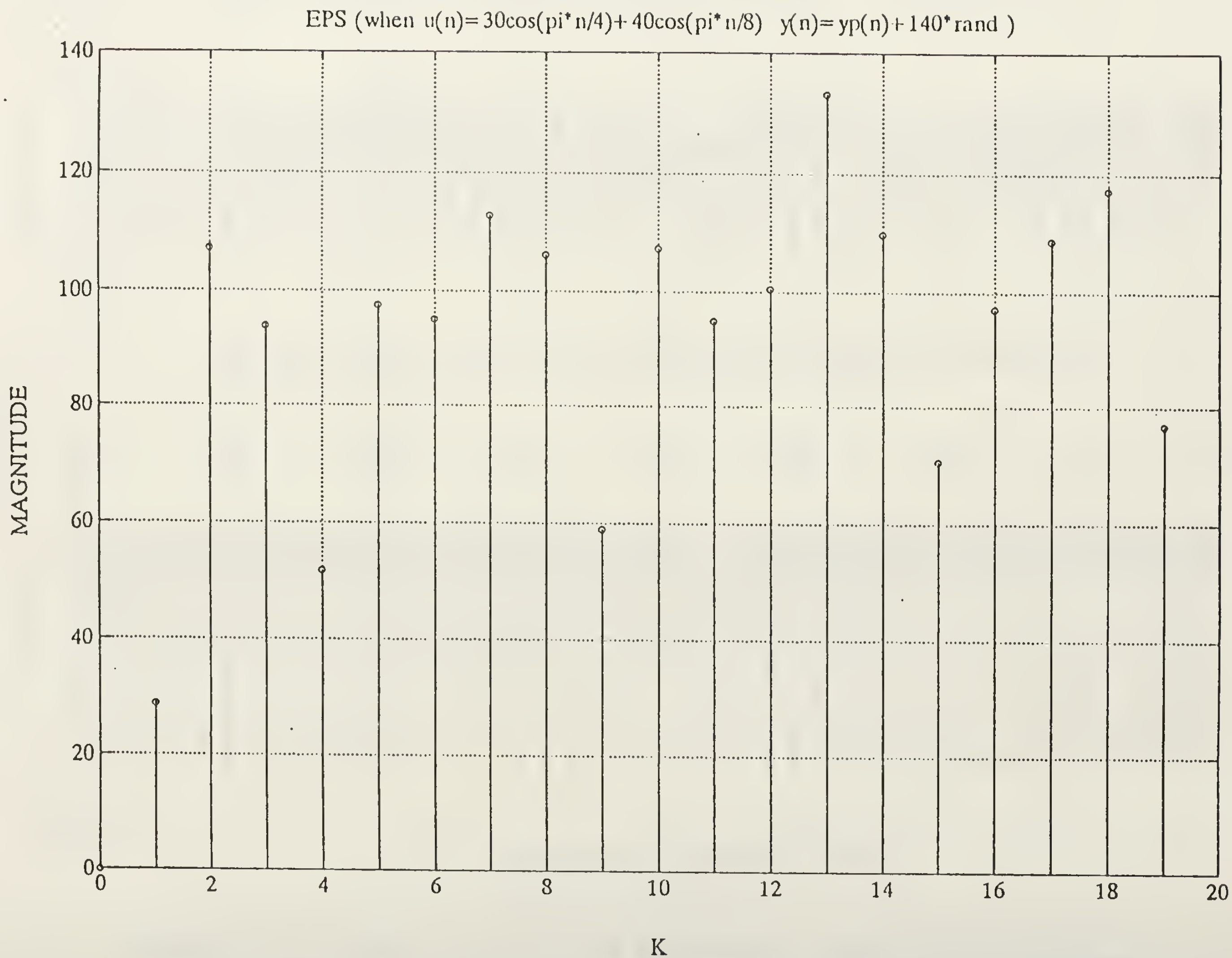

Figure 3.30 Error Between Original and Predicted Output

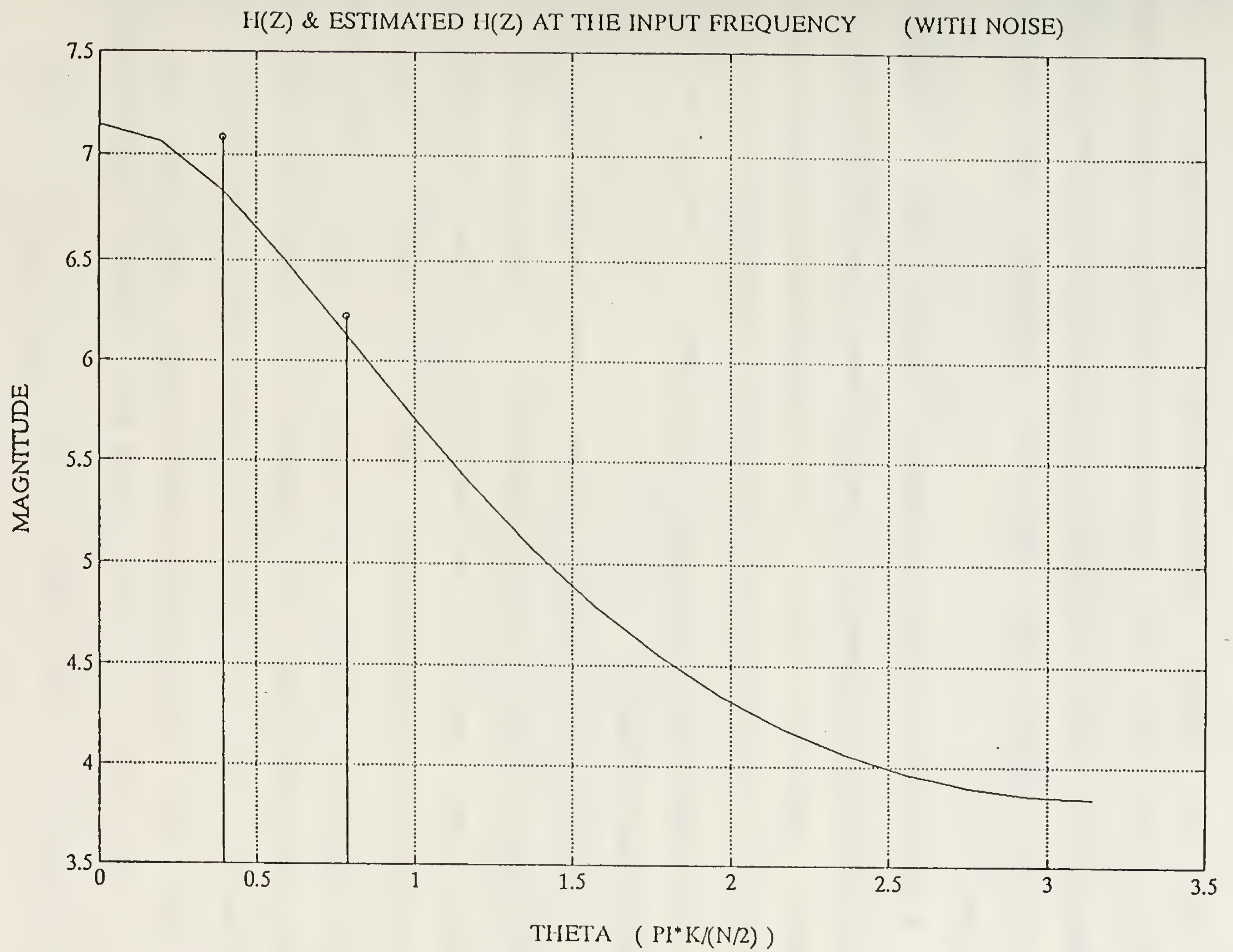

Figure 3.31 The Original and Estimated Frequency Response

IV. CONCLUSION

In this thesis we introduced an approach to the identification of linear models based on the frequency domain formulation. The recursive nature and the use of FFT techniques make the approach attractive for on-line implementation.

A particular aspect we addressed is the identification of the frequency response of the system by approximating the impulse response with a finite sequence. By use of a simulated example we have explored the convergence properties of the algorithm, and its robustness in the presence of measurement noise.

Several issues remain to be investigated, mainly the advantages (if any) of this approach in comparison with more conventional time domain estimation techniques. Although this is subject of future research, as a conjecture we can say that the recursive frequency domain approach might be more robust than the conventional time domain in the cases when the input signal has energy concentrated around a few frequencies, such as the case of periodic excitation. In this case the processing gain of the DFT should give a better performance in the presence of measurement noise.

APPENDIX A : MAINPROGRAM

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                          %
% THESIS MAIN PROGRAM          File name : THESIS4.M       %
%                                                          %
% Advisor : Roberto Cristi                                 %
% Student : Chao, Chi-Shun                                 %
% Date    : Jan, 1, 1991                                   %
%                                                          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Set some constant number

clg
clear
tfinal = 80;                % The max. time to run
dt = 0.1;                   % Sampling time
kmax = tfinal/dt;           % The max. number of signals
rand('normal');             % Set the type of noise
rand_mag = 35;              % Set the gain of noise
M = 20;                     % The # to calculate the error
N = 32;                     % The # of signals in a block
N2= N/2;

% Set the initial condition
yp(1)=0;
yp(2)=0;
y(1)=0;
y(2)=0;
u(1)=0;
u(2)=0;


% Produce the input and coresponding output
% at different frequencies
for n=3:kmax

    %u(n)= 10;
    u(n)= 20*cos(pi*n*5/16);
```

```
    %u(n)= 30*cos(pi*n/4);
    %u(n)= 40*cos(pi*n/8);
    %u(n)= 50*cos(pi*n/16);
    %u(n)= 60*cos(pi*n/32);
    %u(n)= 10+30*cos(pi*n/4);
    %u(n)= 40*cos(pi*n/8)+30*cos(pi*n/4);
    %u(n)= 10+20*cos(pi*n/2)+30*cos(pi*n/4)+40*cos(pi*n/8);
    %u(n)= 10+20*cos(pi*n/2)+30*cos(pi*n/8) ...
           +40*cos(pi*n/8+50*cos(pi*n/16);

    yp(n)=0.3*yp(n-1)+5.0*u(n-1);
    y(n)=yp(n)+rand_mag*rand;

end


% Produce the figures of input and output
plot(u);
title('INPUT U(n) ');
xlabel('n');  ylabel('MAGNITUDE');
grid;
pause;

clg
subplot(211),plot(yp);
title('YP(n) ORIGINAL OUTPUT WITHOUT NOISE ');
xlabel('n');  ylabel('MAGNITUDE');
grid;

subplot(212),plot(y);
title('Y(n) OUTPUT WITH NOISE  u(n)=20cos(5*pi*n/16) ...
       y(n)=yp(n)');
xlabel('n');   ylabel('MAGNITUDE');
grid;
%meta thesis44
pause;
```

```
% Produce the frequency response of the original system
% by using the "dbode" function
num=[5];
den=[1,-0.3];
w=linspace(0,pi,17);
[mag,phase]=dbode(num,den,w);

clg
subplot(211),plot(w,mag);
title('FREQUENCY RESPONSE OF THE ORIGINAL SYSTEM H(Z)');
xlabel('THETA');
ylabel('MAGNITUDE');
grid;

subplot(212),plot(w,phase);
title('FREQUENCY RESPONSE OF THE ORIGINAL SYSTEM H(Z)');
xlabel('THETA');
ylabel('PHASE (DEGREE)');
grid;
pause;


%  Create the matrix H
v=[zeros(1,N2),ones(1,N2)];
h=diag(v);
F=fft(eye(N));
H=F*h*inv(F);


% Set the zero matrix
THETAK = zeros(N,1);
P0=10*eye(N);
P=P0;
```

```
% Use the FFT and RLS (Recursive Least Square) techniques
% to estimate frequency response of the model. It is an
% interactive and iterative loop
for k = 2 : M
      yk0 = y( (k-1)*N2+1 : k*N2 )';
      uk = u( (k-2)*N2+1 : k*N2 )';
      yk=[zeros(N2,1);yk0];
      YK = fft( yk );
      UK = fft( uk );

      EPS(k)=0.0;

      for i = 1 : N

            PHI=diag(H(i,:))*UK;

            [THETAK,P]=rls(THETAK,P,PHI,YK(i));

            EPS(k)=EPS(k)+(YK(i)-conj(PHI')*THETAK)'* ...
                    (YK(i)-conj(PHI')*THETAK);
      end


      THETA(:,k) =  THETAK ;

end

EPS = sqrt(EPS)/N;

% Calculate the corresponding horizontal axis
for k=0:N/2
      w1(k+1)=2*pi*k/N;
end
```

```
% Produce the frequency response figures of estimated model
clg
subplot(211),plot(w1,abs(THETA(1:17,M)));
title('EST. H(Z) ');
xlabel('THETA    ( PI*K/(N/2) )');
ylabel('MAGNITUDE');
grid;

subplot(212),plot(w1,angle(THETA(1:17,M)));
title('EST. H(Z) ');
xlabel('THETA     ( PI*K/(N/2) )');
ylabel('PHASE (RADIAN)');
grid;
pause;


% Produce the error figures
clg
displot(EPS);
title('EPS (when u(n)=20cos(5*pi*n/16)  ...
       y(n)=yp(n)+35*rand  )');
xlabel('K');
ylabel('MAGNITUDE');
grid;
%meta thesis44
pause;


% Produce the frequency response of the estimated model
% in a continuous pattern.
clg
plot(w,mag,w1,abs(THETA(1:17,M)));
title('H(Z) & EST. H(Z) ');
xlabel('THETA   ( PI*K/(N/2) )');
ylabel('MAGNITUDE');
grid;
pause;
```

```
% Store the value of the response of the estimated model
% at the input frequency.
THETA_F = zeros(17,1);
%THETA_F(1,1)  = THETA(1,M);
%THETA_F(2,1)  = THETA(2,M);
%THETA_F(3,1)  = THETA(3,M);
%THETA_F(5,1)  = THETA(5,M);
%THETA_F(9,1)  = THETA(9,M);
THETA_F(6,1)  = THETA(6,M);


% Produce the frequency response of the original system and
% the response of estimated model at the input frequency
% in the same figure.
clg
plot(w,mag);
title('H(Z) & ESTIMATED H(Z) AT THE INPUT FREQUENCY ...
       (WITH NOISE)');
xlabel('THETA   ( PI*K/(N/2) )');
ylabel('MAGNITUDE');
grid;
hold on
displot(w1,abs(THETA_F));
hold off
%meta thesis44
```

APPENDIX B : FUNCTION TYPE SUBROUTINE RLS

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                          %
% SUBROUTINE TYPE FUNCTION         File name : RLS.M       %
%                                                          %
% Advisor : Roberto Cristi                                 %
% Student : Chao, Chi-Shun                                 %
% Date    : Jan, 1, 1991                                   %
%                                                          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [x,p]=rls(x,p,phi,y)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                          %
%  It updates the estimate x and its covariance p          %
%  by standard recursive least squares.                    %
%                                                          %
%  data:    x = column vector (input, output)              %
%                                                          %
%      p = square matrix, positive definite(input,output)  %
%      y = scalar (input)                                  %
%      phi = column vector (input)                         %
%                                                          %
%  NOTE:  Due to numerical approximations it might be      %
%         a good idea to check the matrix p for positive   %
%         definiteness.                                    %
%                                                          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

      fact = 1.0 + phi'*p*phi;
      x = x + p*conj(phi)*(y-conj(phi')*x)/fact;
      p = p - p*conj(phi)*conj(phi')*p/fact;
% end rls
```

LIST OF REFERENCES

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2. Karl J. Astrom and Bjorn Wittenmark, "Computer Controlled Systems Theory and Design", Prentice-Hall, Inc., pp. 324-338, 1984.

3. David Mansour and Augustine H. Gray, JR., "Unconstrained Frequency Domain Adaptive Filter", IEEE Transactions on Acoustics, Speech and Signal Processing, Vol. ASSP-30, No. 5, pp. 726-734, October 1982.

4. Howard Elliott and Roberto Cristi and Manohar Das, "Global Stability of Adaptive Pole Placement Algorithm", IEEE Transaction on Automatic Control, Vol AC-30, No. 4, pp. 348-356, Appril 1988.

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Thesis
C3735 Chao
c.1 A frequency domain based approach to on-line identification.